AUPEC 2017
Tutorial

Principles and Practices of Digital Current Regulation for AC Systems

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Summary

- Power electronic AC conversion systems require precise current regulation with high bandwidth

- Conventional wisdom:
  - Stationary Frame PI regulator gains can never be set high enough to get acceptably small steady state error
  - Synchronous Frame (P+Resonant) control is mandatory

- Unresolved question:

  What are the stationary frame PI gain limits? - simple theory says they can be arbitrarily large without regulator instability

- Answering this question allows best possible PI regulator performance to be determined for any particular application

Primary Objectives of a Current Regulation System

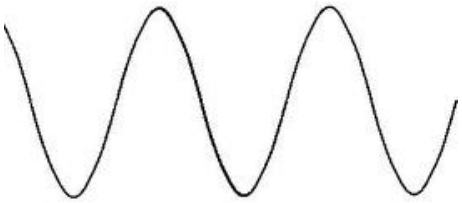
- Minimize steady state magnitude (and phase for an AC system) output variable error, preferably achieving zero steady state error.
- Accurately track the commanded reference variable during transient changes. This requires a regulation system to have as high a bandwidth as possible, to achieve the best possible dynamic response.
- Limit the peak value of the output variable, to avoid overload conditions.
- Minimize low order harmonics in the load variable, and compensate for DC-link voltage ripple, deadtime delays, semiconductor device voltage drops, load parameter variation and other practical second order effects associated with the operation of the electrical conversion system.

Introduction

Electrical Source

Switched Power Converter

Load Motor



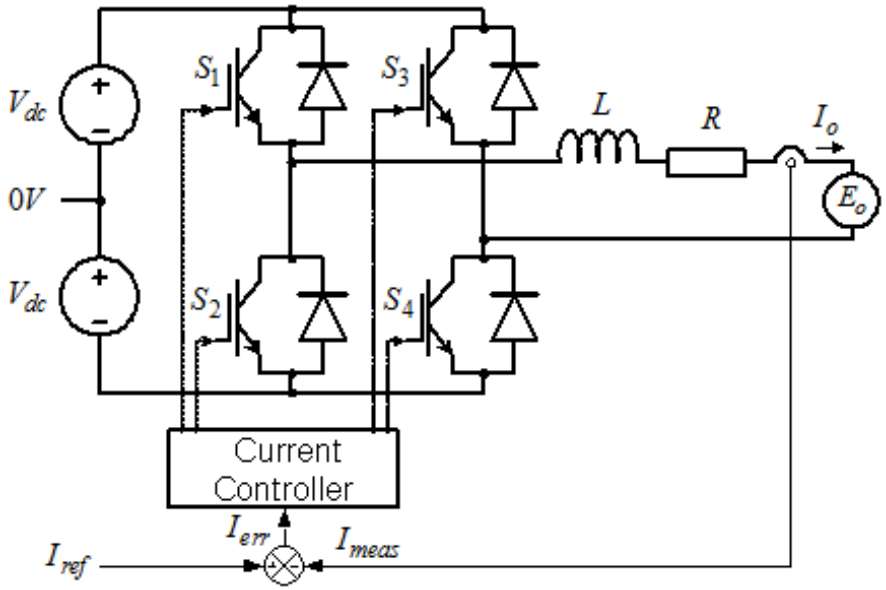
Target Current Reference

Controller

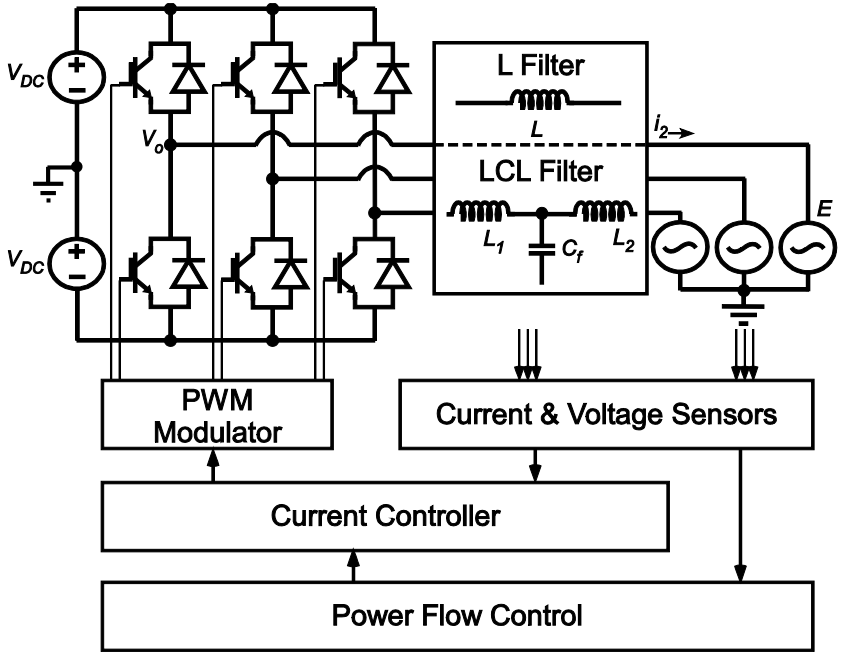
Current Measurement

General Arrangement of Current Regulation for an Electrical Motor Drive System

Introduction



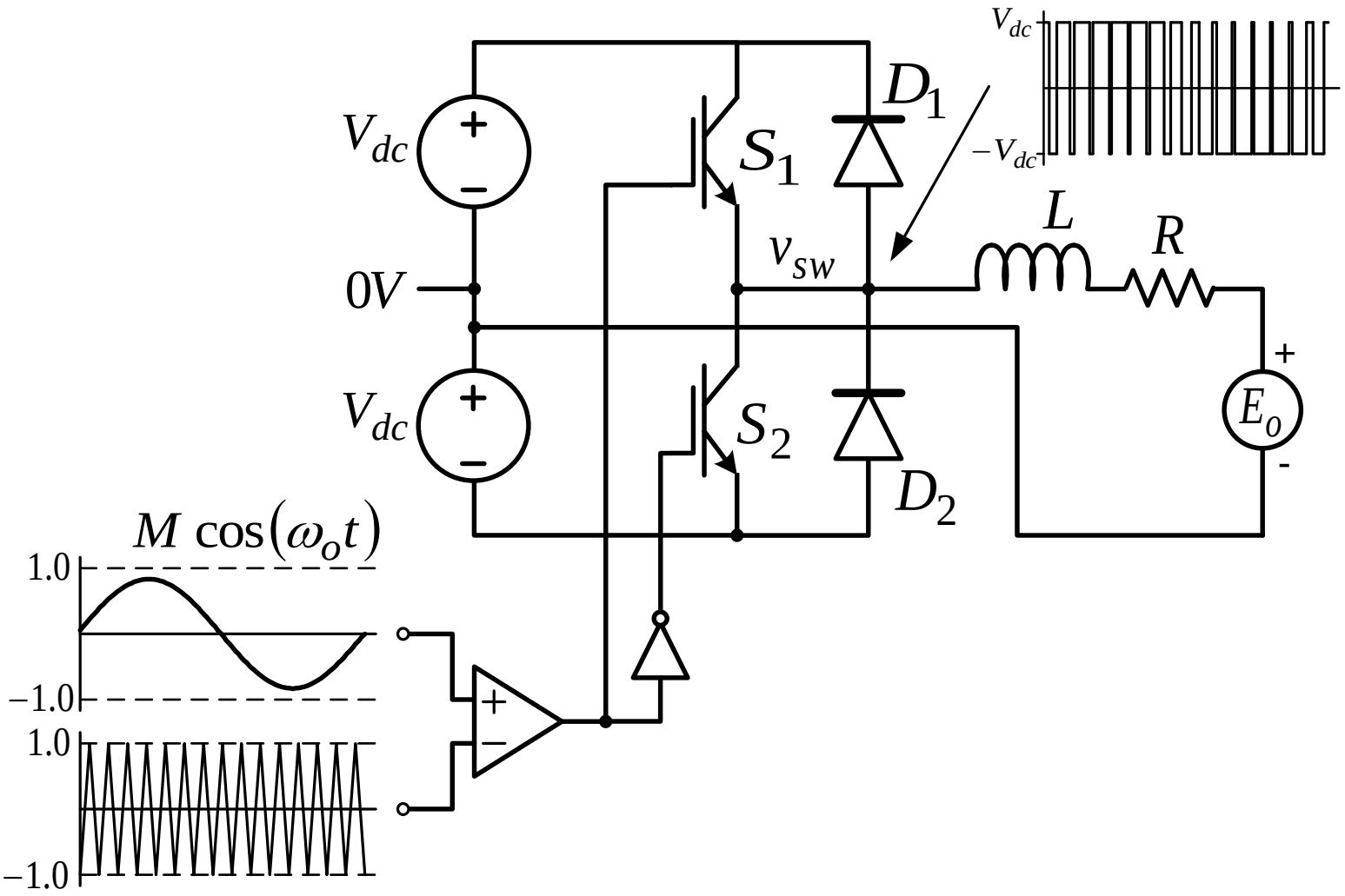
Single Phase Current Regulated System



Three Phase Current Regulated System (L or LCL Filter)

Electrical Schematic of AC Current Regulated Systems

Principle of Pulse Width Modulation



W.R. Bennett:1933 H.R. Black: 1953 S. Bowes: 1975

Pulse Width Modulation of Voltage Source Inverter

Analytical Solution is obtained by Double Fourier Analysis of the Switched Phase Leg Output Waveform:

$$f(t) = \frac{A_{00}}{2} + \sum_{n=1}^{\infty} [A_{0n}\cos(n[\omega_o t + \theta_o]) + B_{0n}\sin(n[\omega_o t + \theta_o])]$$

DC Offset

Fundamental Component & Baseband Harmonics

$$+ \sum_{m=1}^{\infty} [A_{m0}\cos(m[\omega_c t + \theta_c]) + B_{m0}\sin(m[\omega_c t + \theta_c])]$$

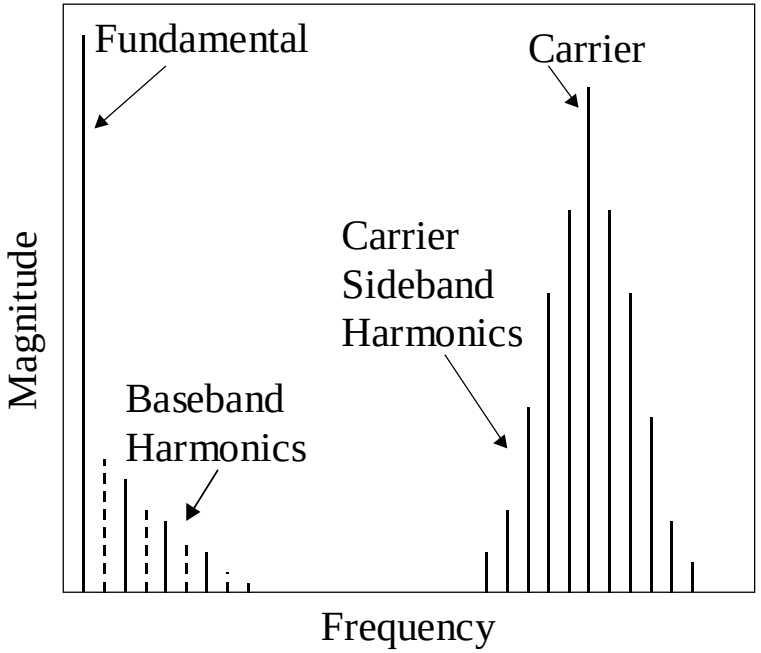
Carrier Harmonics

$$+ \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ (n \neq 0)}}^{\infty} \left[\begin{array}{l} A_{mn}\cos(m[\omega_c t + \theta_c] + n[\omega_o t + \theta_o]) \\ + B_{mn}\sin(m[\omega_c t + \theta_c] + n[\omega_o t + \theta_o]) \end{array} \right]$$

Sideband Harmonics

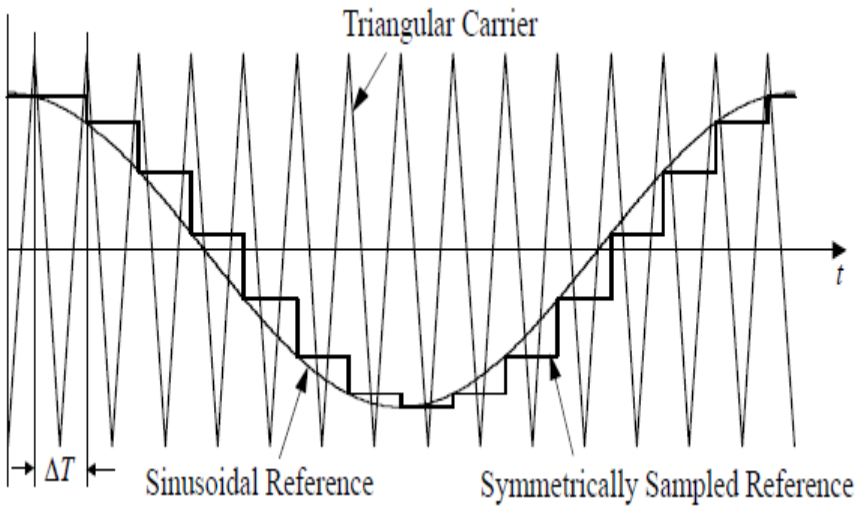
The solution contains:

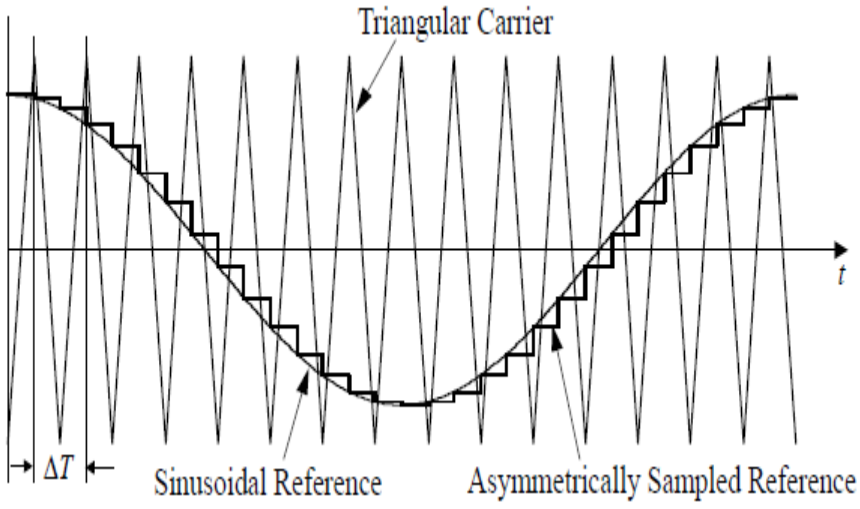
- Fundamental component
- Baseband Harmonics
- Carrier Harmonics
- Sideband Harmonics



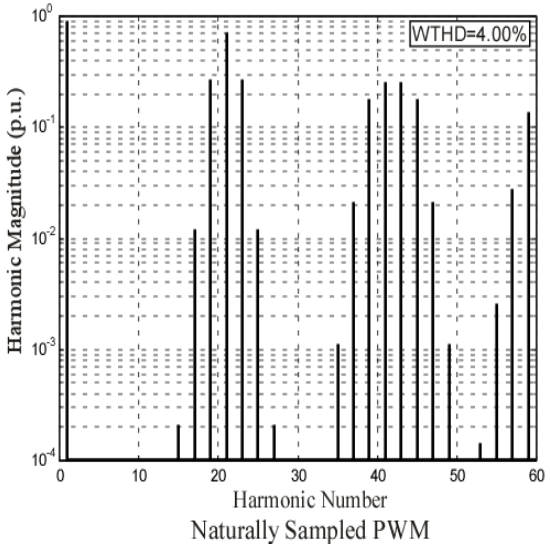
Fundamental and Baseband Harmonics are the relevant harmonics for current regulation.

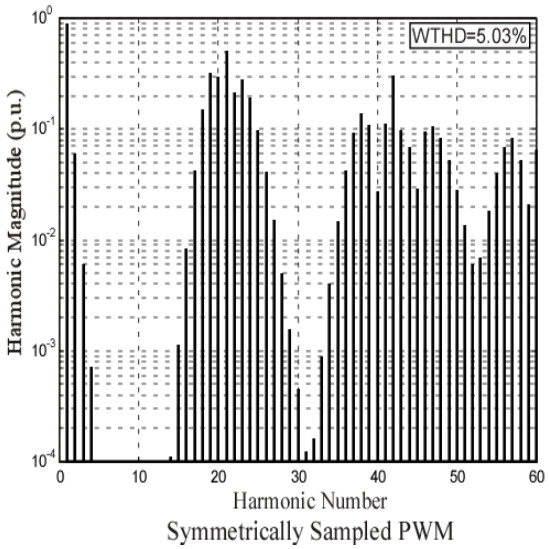
Sampled PWM of Inverter Phase Leg

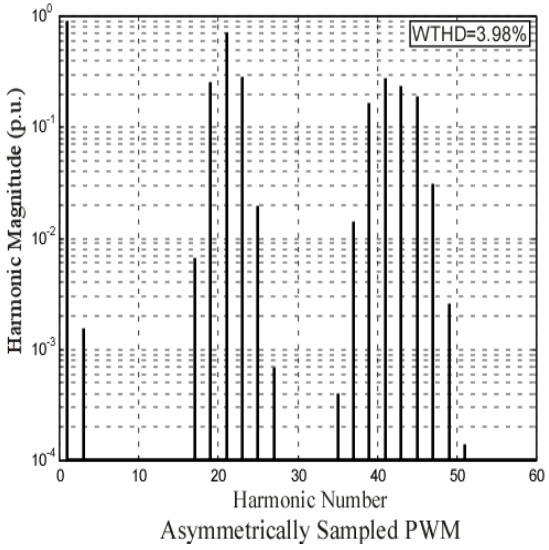




NOTE: Sampled references are delayed wrt their sinusoidal sources.

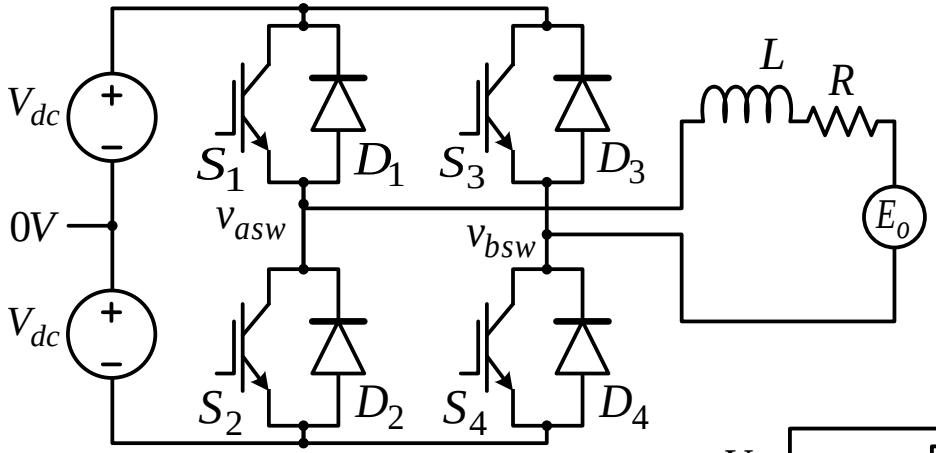






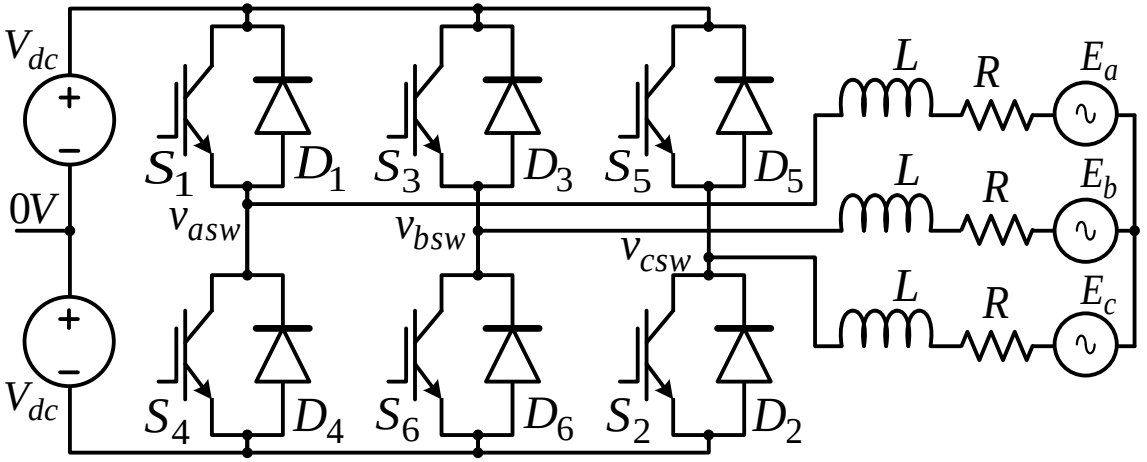
Two Level Converter Topologies

Combining Phase Legs creates the well known single phase and three phase Voltage Source Inverters,



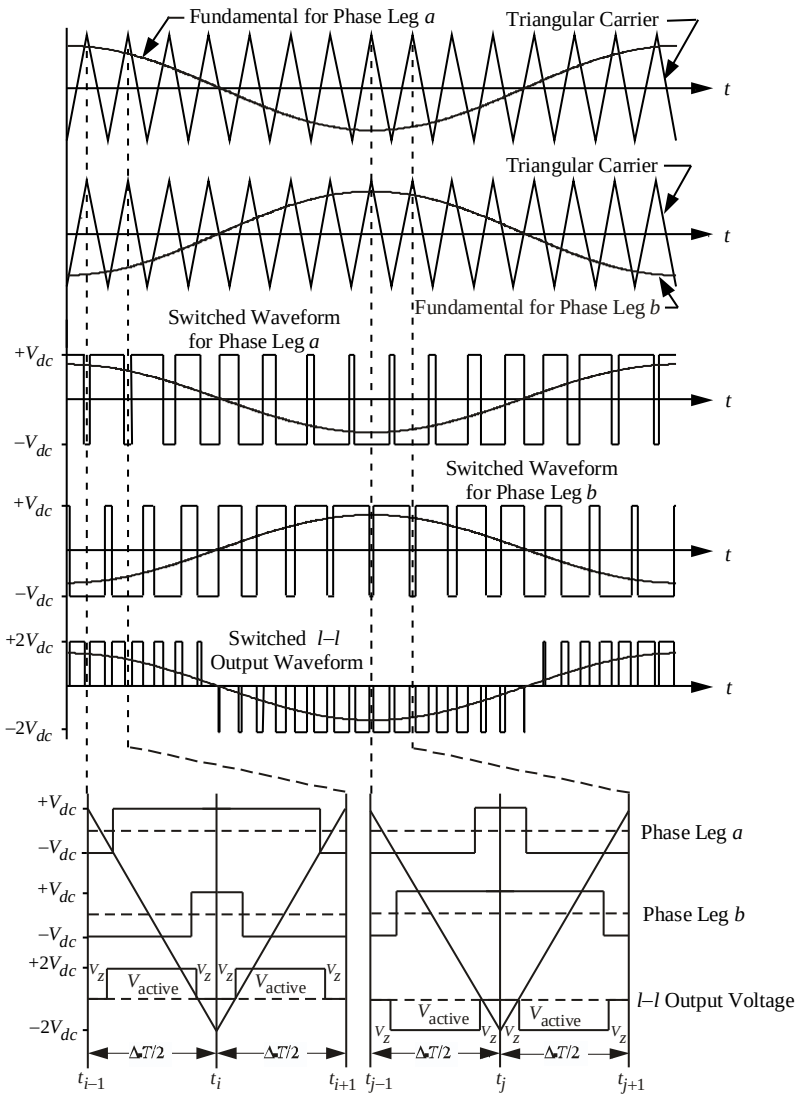
Single Phase VSI

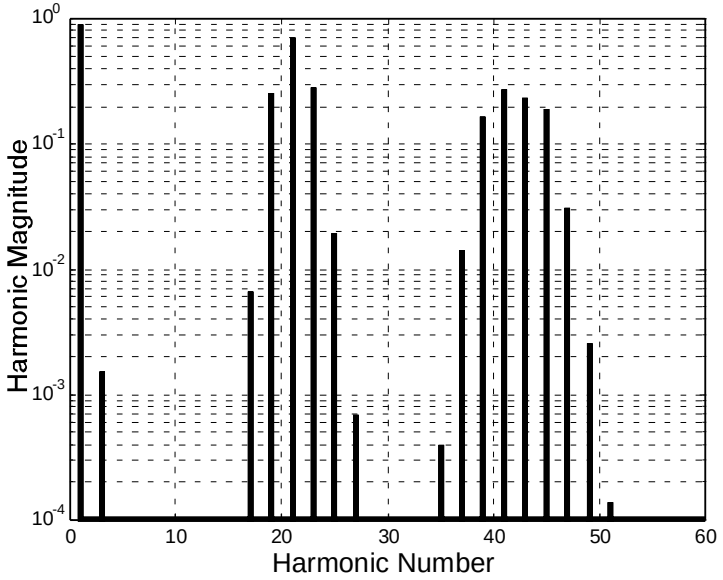
which cancel various harmonics between the phase legs depending on the modulation strategy, i.e.

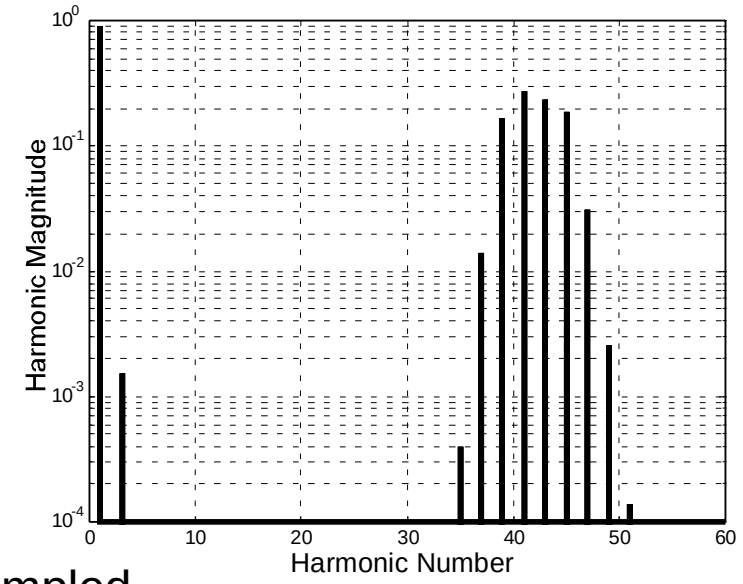


Three Phase VSI

PWM of Single Phase VSI

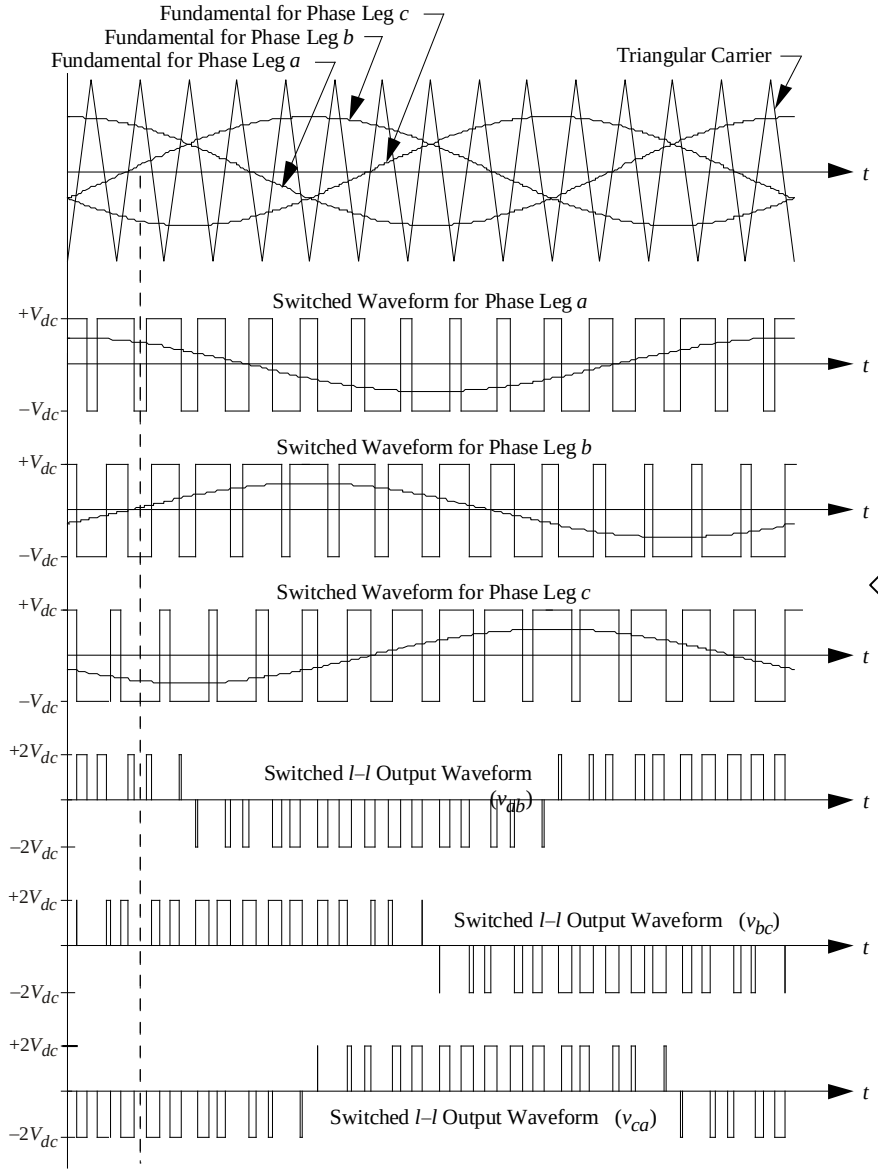


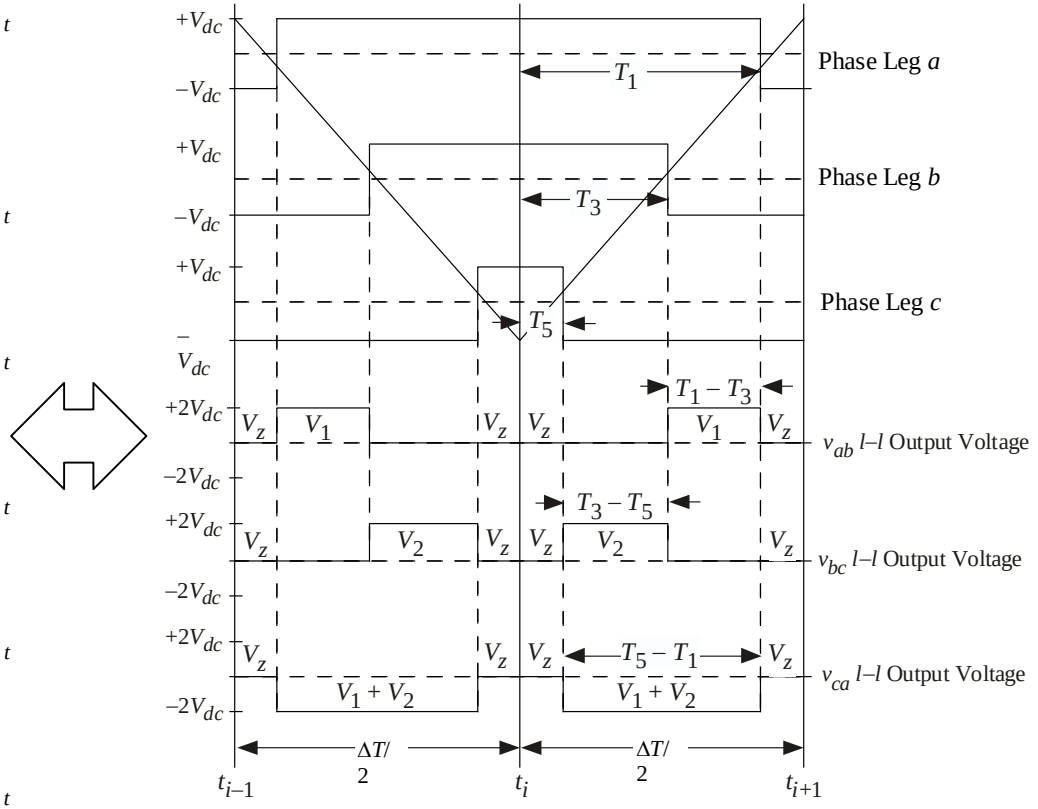




Single Phase Three level Continuous PWM - sampled

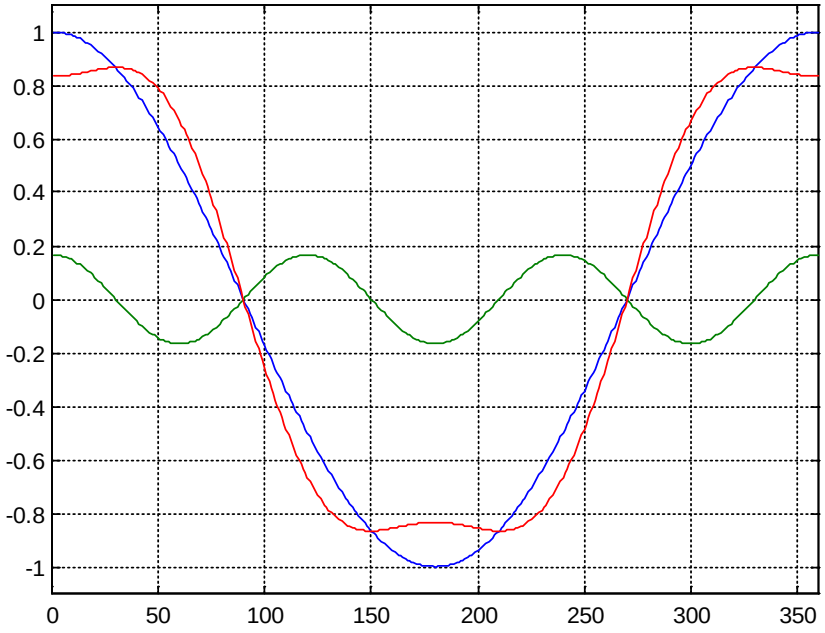
PWM of Three Phase VSI



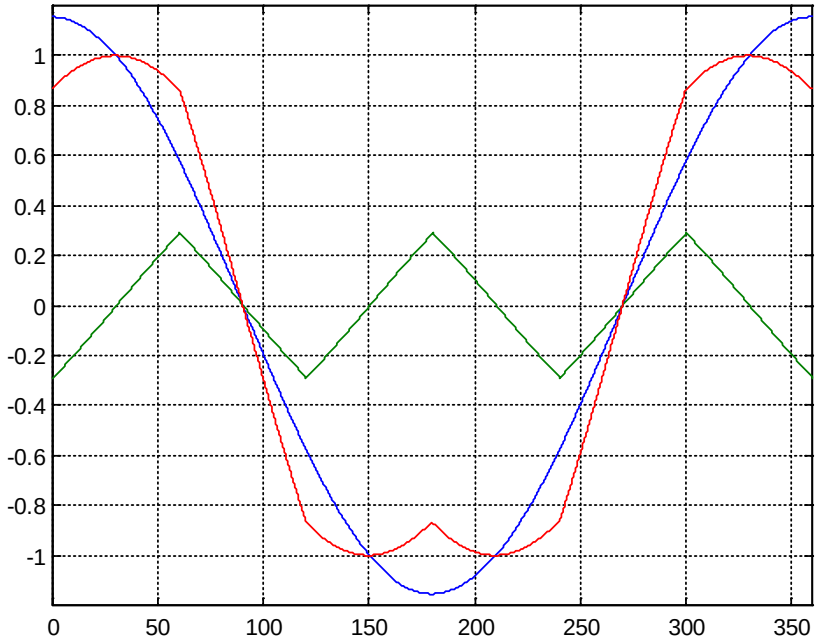


Third Harmonic Injection/Space Vector Modulation

Third harmonic injection and Space Vector modulation allow an increase in modulation index without reaching saturation limits.



Third Harmonic Injection



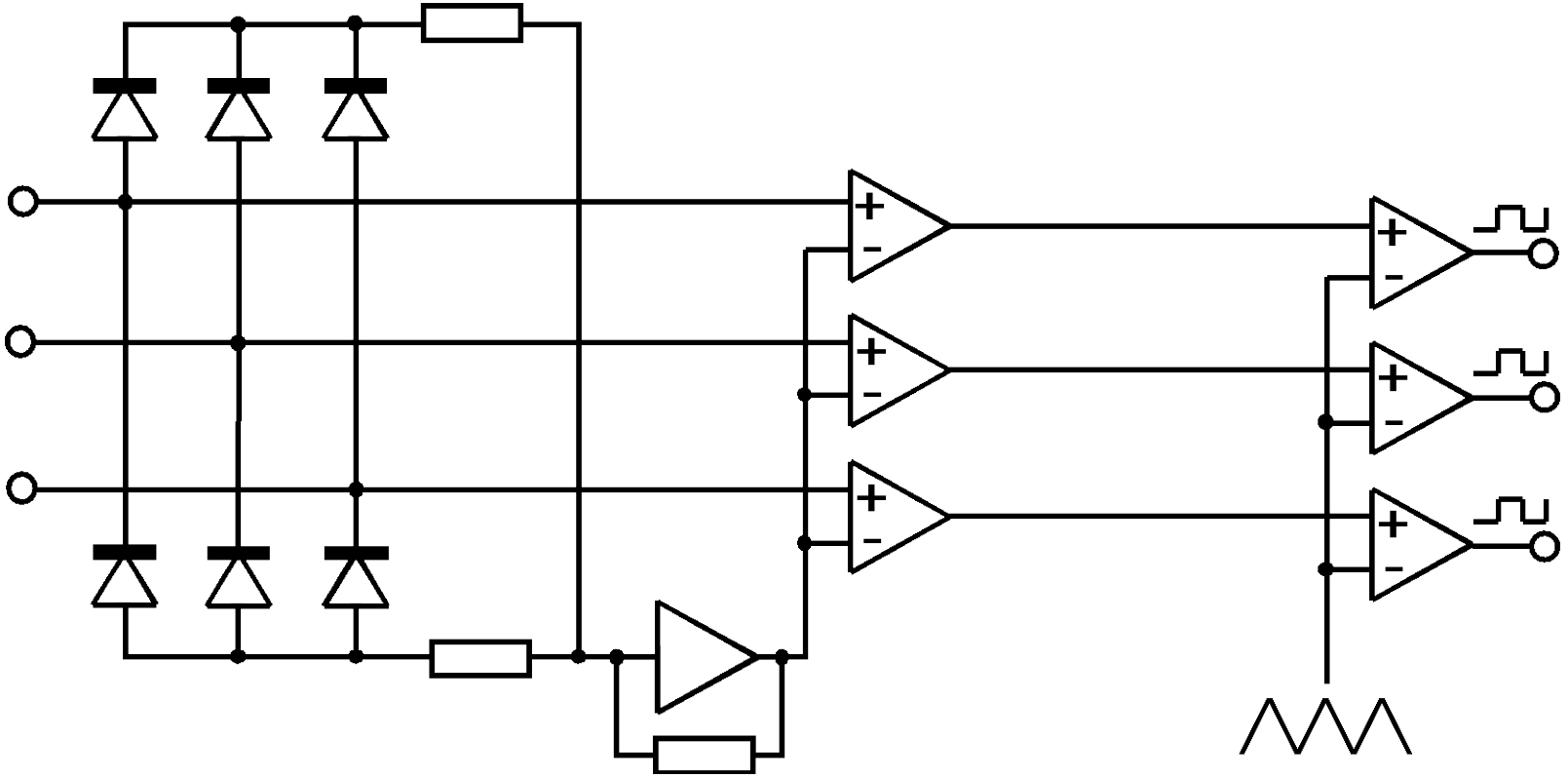
Space Vector Modulation

Pulse Width Modulation

Common Myths about PWM:

- Must have odd carrier ratio to avoid even harmonics
- Must have triplen carrier ratio for three phase system
- Space Vector Modulation is superior to Sine-triangle Modulation
- Instantaneous Third Harmonic Injection is impossible
- Analog SV modulation is impossible

Instantaneous SV/3rd Harmonic Implementation

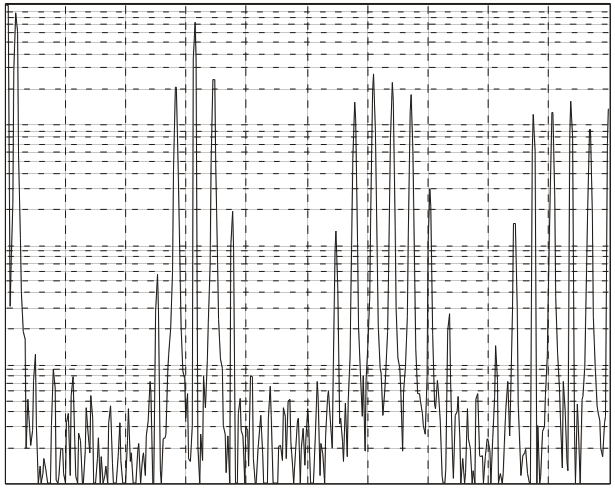


Analog Implementation of SV modulator

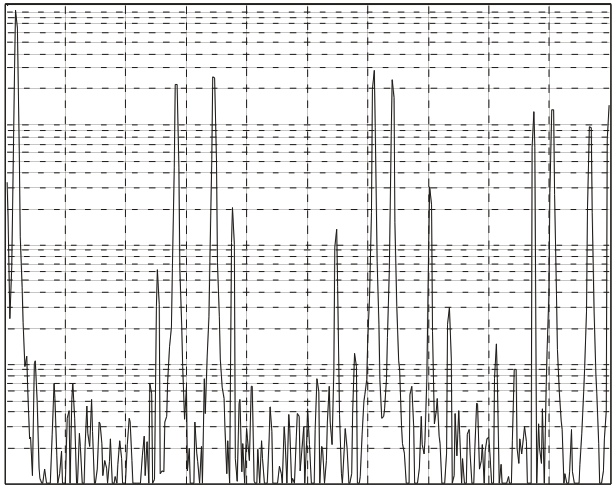
$$v_{off} = \frac{\max(v_a, v_b, v_c) + \min(v_a, v_b, v_c)}{2}$$

Digital Implementation of SV modulator

PWM of Three Phase Voltage Source Inverter

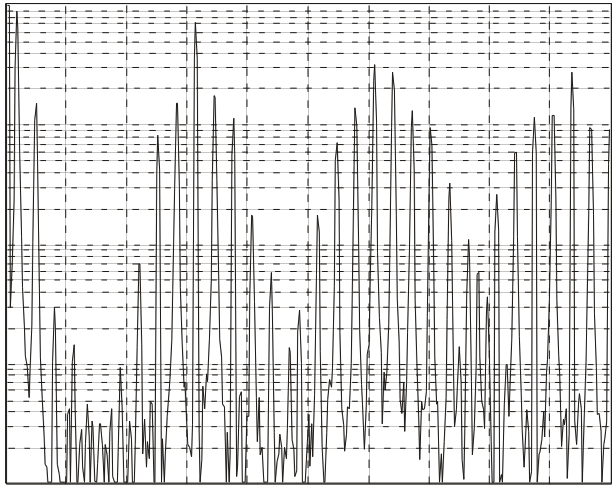


Phase Leg

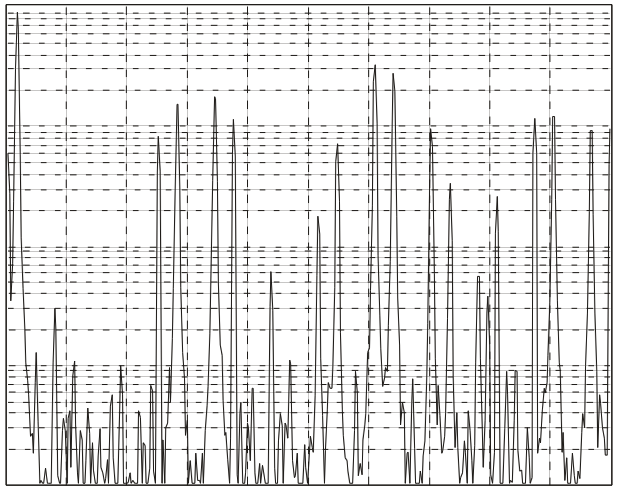


L-L Voltage

Sinusoidal Reference



Phase Leg



L-L Voltage

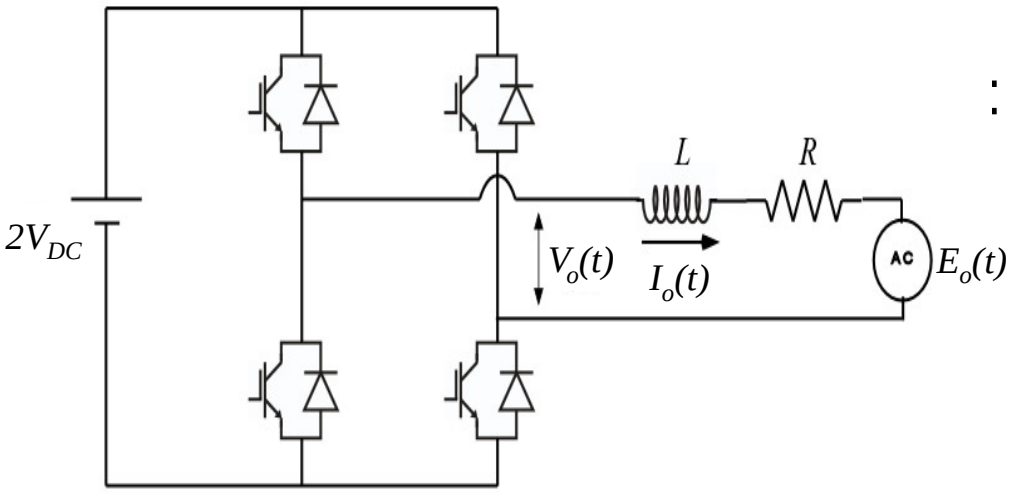
Sine+ 3rd Harmonic

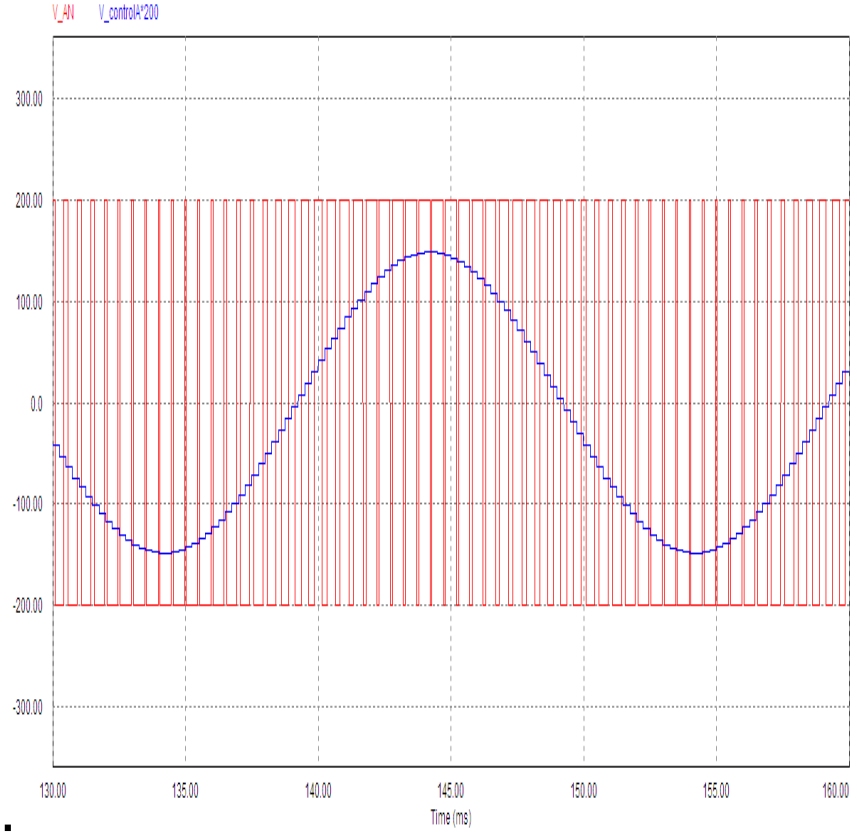
Experimental Confirmation of Theoretical PWM Harmonics

Linear Current Regulation

- Ideal “Average” Linear Model
- Transport and Sampling Delay Limitations
- Maximum Gain Determination – backemf compensation, Proportional-Resonant, d-q synchronous frame
- Three Phase Current Regulation
- Discrete Time Domain Models
- Current Regulation with LCL Filters

Ideal “Average” Model of Single Phase VSI





$$V_o(t) - E_o(t) = RI_o(t) + L\frac{dI_o(t)}{dt}$$

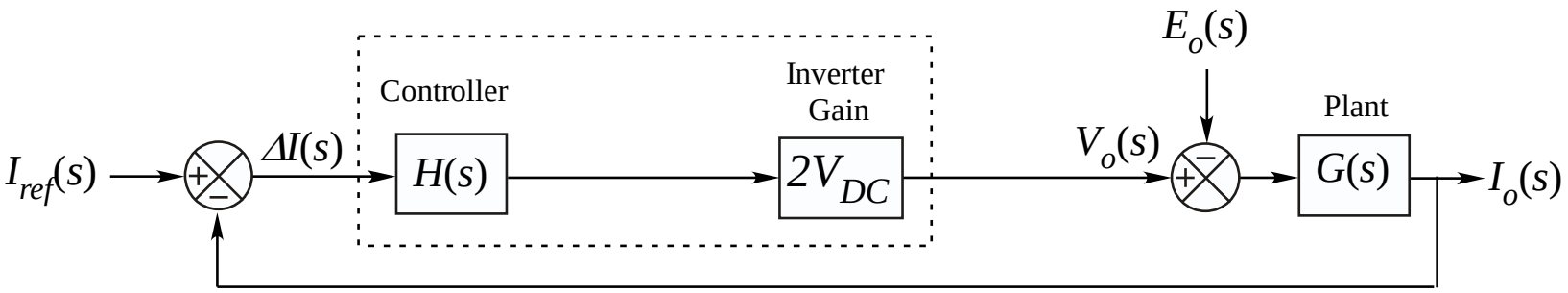
- Averaged model in Laplace form:

$$V_o(t) = M(t)2V_{DC}$$

$$I_o(s) = \frac{V_o(s) - E_o(s)}{R + sL}$$

Ideal Linear AC Current Regulated System

The ideal system reduces (in control terms) to the s-domain form of:



under the simplifications of

- The Modulator/VSI reduces to a linear gain $(2V_{DC})$, ignoring switching harmonics (which cannot be controlled anyway)
- The input disturbance (E_o) represents the motor back-emf
- Plant block is defined by:

$$G(s) = \frac{1}{R + sL} = \frac{1}{R} \cdot \frac{1}{1 + sT_p}$$

$$T_p = \frac{L}{R}$$

Ideal Linear AC Current Regulated System

- Analysis of the system block diagram:

$$\begin{aligned} I_o(s) &= G(s)\left[V_o(s) - E_o(s)\right] \\ &= G(s)H(s)2V_{DC}\left[I_{ref}(s) - I_o(s)\right] - G(s)E_o(s) \end{aligned}$$

$$I_o(s)\left[1 + G(s)H(s)2V_{DC}\right] = G(s)H(s)2V_{DC}I_{ref}(s) - G(s)E_o(s)$$

- Solving for $I_o(s)$ gives:

$$I_o(s) = I_{ref}(s)\frac{H(s)2V_{DC}G(s)}{1 + H(s)2V_{DC}G(s)} - E_o(s)\frac{G(s)}{1 + H(s)2V_{DC}G(s)}$$

- This is the closed loop transfer function of the system

Ideal Linear AC Current Regulated System

- This transfer function includes two terms:
  - Reference tracking component

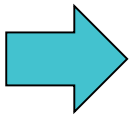
$$\frac{I_o(s)}{I_{ref}(s)} = \frac{H(s)2V_{DC}G(s)}{1+H(s)2V_{DC}G(s)}$$

  - Disturbance rejection:

$$\frac{I_o(s)}{E_o(s)} = -\frac{G(s)}{1+H(s)2V_{DC}G(s)}$$

- Primary performance objectives are:

$$\frac{I_o(s)}{I_{ref}(s)} \approx 1 \qquad \frac{I_o(s)}{E_o(s)} \leq \varepsilon$$



Make H(s) as large as possible

Ideal Linear AC Current Regulated System

- Try a simple PI regulator:

$$H(s) = K_P\left(1+\frac{1}{s\tau_i}\right) = K_P\left(\frac{1+s\tau_i}{s\tau_i}\right)$$

- Open Loop Forward Path Gain:

$$H(s)V_{DC}G(s) = \frac{2V_{DC}K_P}{R\tau_i}\cdot\frac{1+s\tau_i}{s(1+sT_p)}$$

- Classical control theory states system is stable provided:

$$\angle 2V_{DC}H(j\omega_c)G(j\omega_c) > -180^\circ \quad when \quad \left|2V_{DC}H(j\omega_c)G(j\omega_c)\right| = 1$$

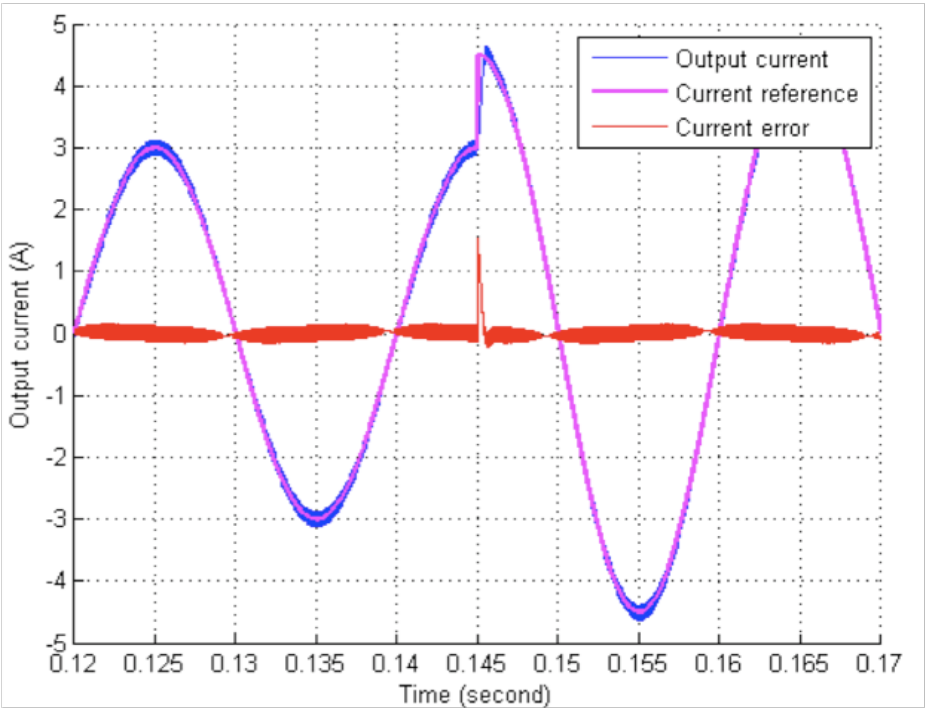
- Simple PI forward path phase response asymptotes to -90^o at high frequencies. Hence system is unconditionally stable irrespective of PI gain K_p and integrator time constant τ_i.

THIS CLEARLY DOES NOT MATCH REALITY ????

Ideal Linear AC Current Regulated System

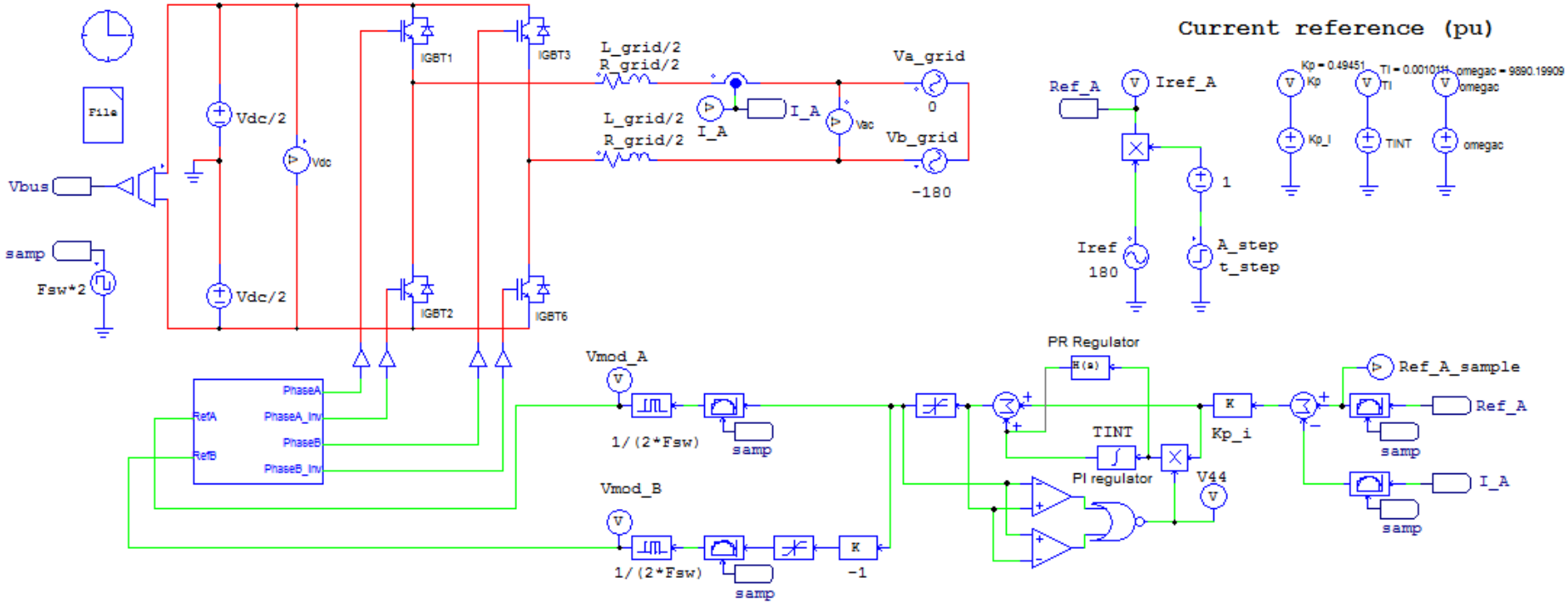
Circuit Parameter	Value
Resistive load (R) (Ω)	2
Inductive load (*L*) (mH)	20
Switching Freq. (f_s) (kHz)	5.0
DC Bus volt. ($2V_{DC}$) (V)	400
Back EMF volt. (V_{EMF}) (V_{RMS})	80
Back EMF freq. (Hz)	50
Sampling period (T) (sec)	10^{-4}

Test circuit parameters



Simulated response of ideal average value model AC current regulator with very high PI gains.

Practical Linear AC Current Regulated System

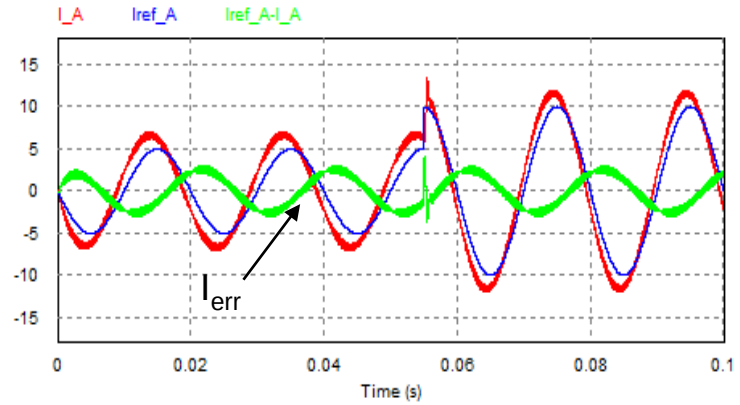


PSIM Current Regulator Simulation with PWM and Controller Delays

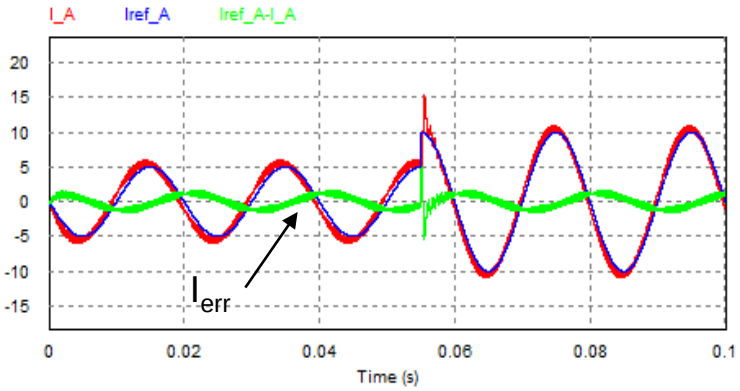
Practical Linear AC Current Regulated System

Circuit Parameter		Value
Resistive load	(R) (Ω)	1.2
Inductive load	(*L*) (mH)	10.0
Switching Freq.	(f_s) (kHz)	5.0
DC Bus volt.	($2V_{DC}$) (V)	400
Back EMF volt.	(V_{EMF}) (V_{RMS})	220
Back EMF freq.	(Hz)	50
Sampling period	(T) (sec)	10^{-4}

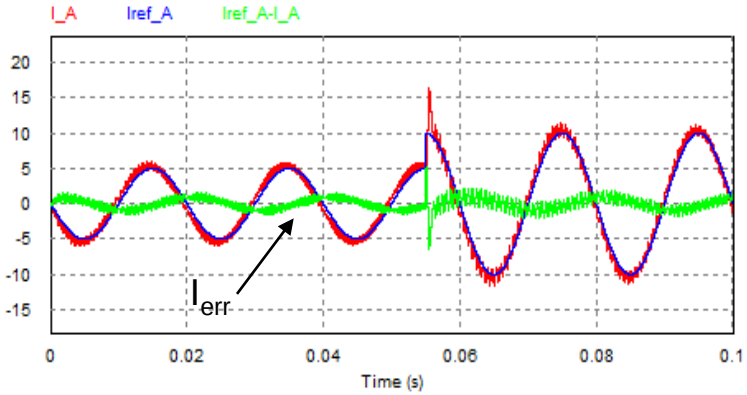
- Clearly gain cannot be arbitrarily increased despite prediction of ideal theory.
- Need to identify the reason, so that PI gains can be set to maximum possible.



K_p = 0.145
Steady state error, damped transient



K_p = 0.218
Reducing steady state error, underdamped transient



K_p = 0.247
Less steady state error, very underdamped transient

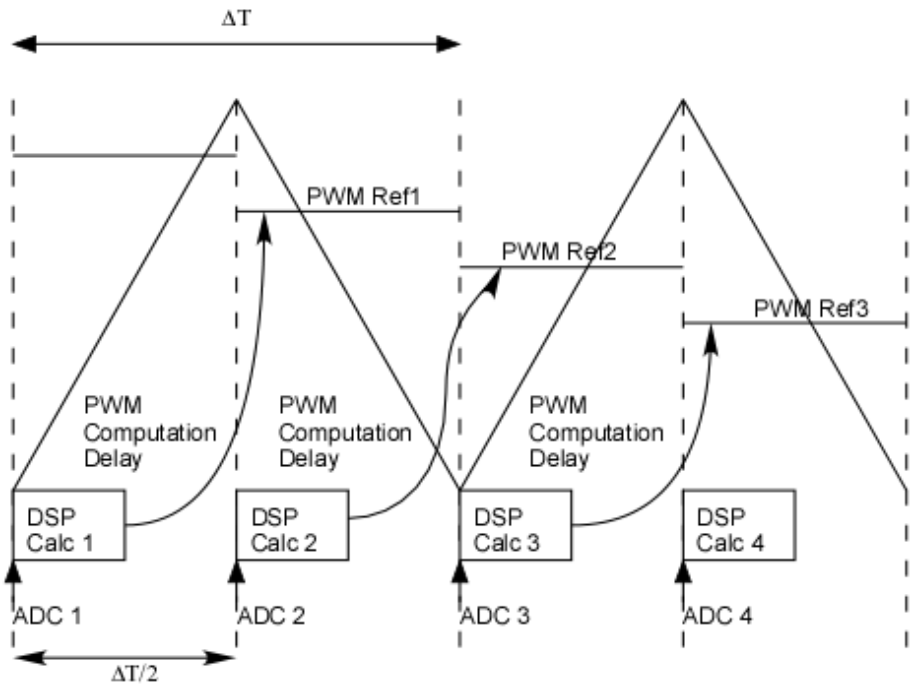
Practical Linear AC Current Regulated System

- Average model must be extended to incorporate 2nd order effects which constrain practical controller gains.

- The two most likely candidates are:
  - Transport delay due to the PWM process
  - Sampling delay caused by digital controller

Sampling and Transport Delay

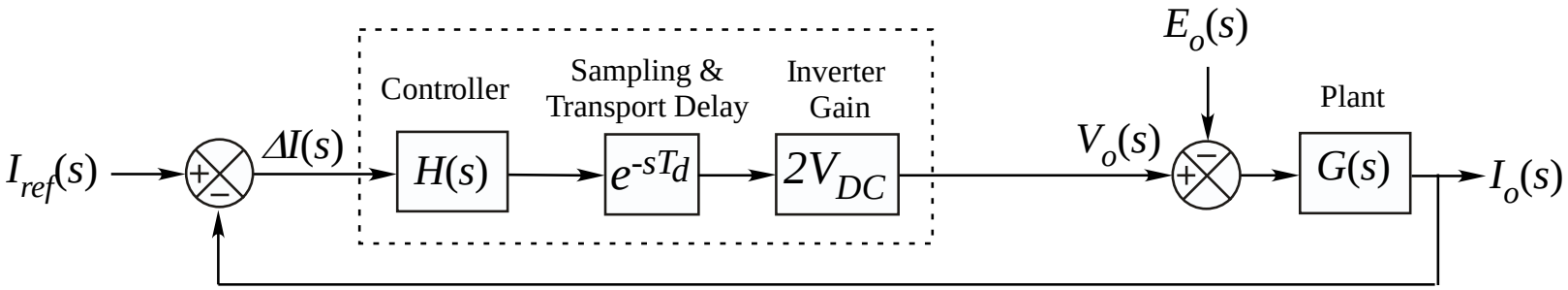
- Asymmetrical regular sampled PWM introduces a quarter carrier-period sampling delay (ZOH) into the control process.
- Computation delay and synchronous sampling cause a further half-carrier transport delay (z^{-1}).
- Overall delay of 0.75 of the carrier period is introduced into the forward path of the control loop.



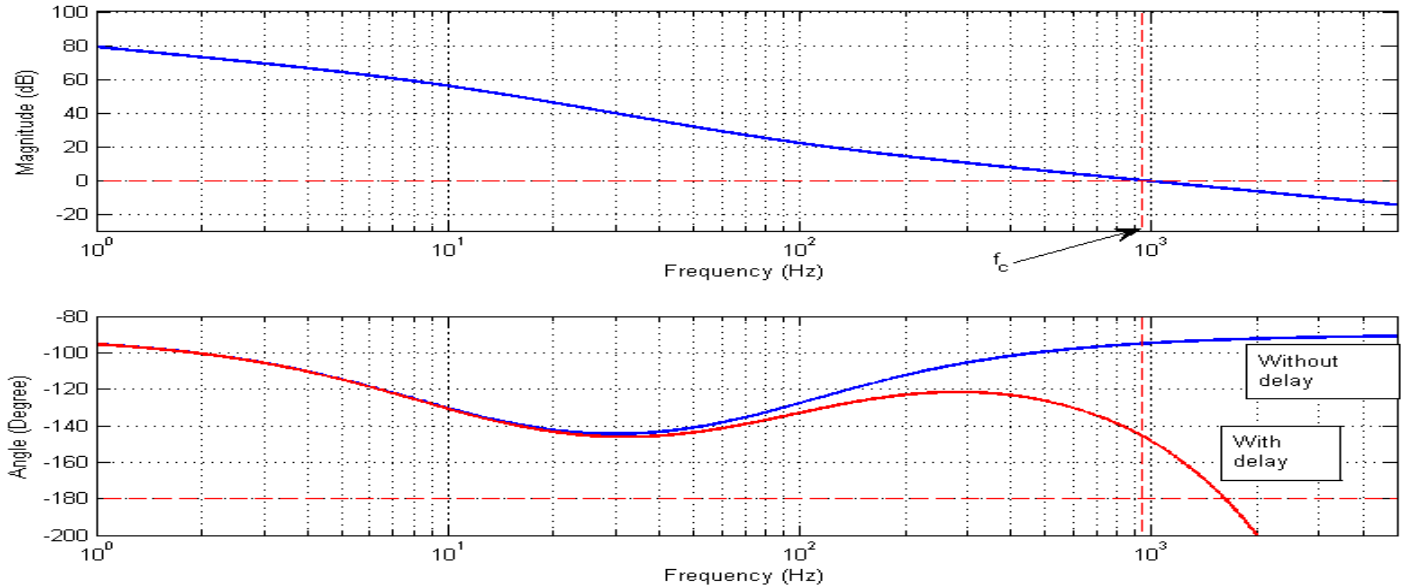
Sampling and Transport Delay caused by PWM process and digital controller sampling/computation.

$$T_d = 0.75\Delta T = 0.75 / f_s$$

Effect of Regulator Delay on Frequency Response



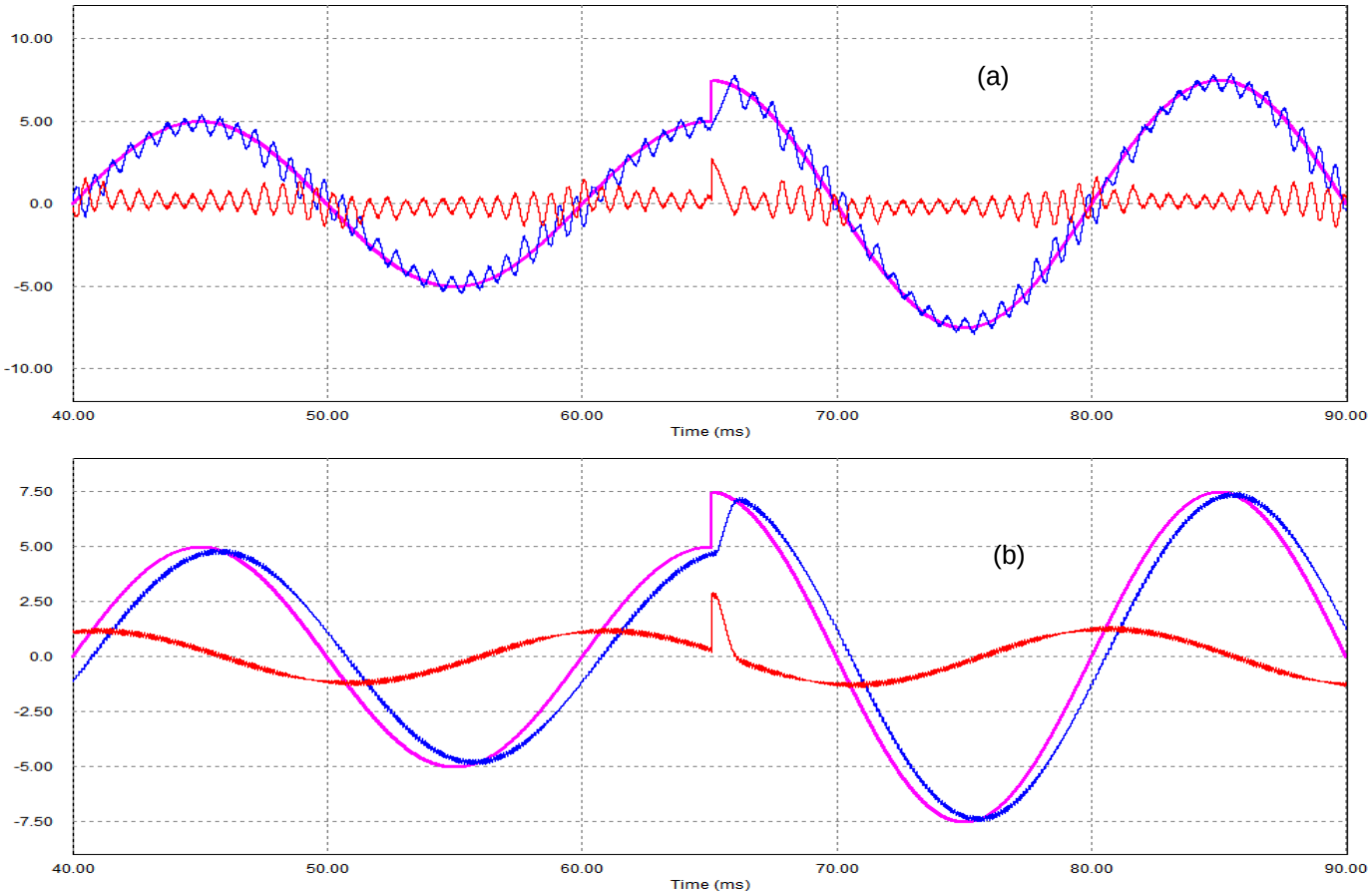
Stationary frame PI Regulator with sampling and transport delay



Bode plot of open loop forward path gain for ideal and regulators with transport delay

The gain limitation is now clear – forward path phase goes below 180^0 as frequency increases. Forward loop gain magnitude must fall less than 1.0 before this occurs.

PI Regulator Response with Regulator Delay



Simulated stationary frame PI regulator with regulator delay
(a) "ideal" high gains (b) gains reduced to maintain stability

WHICH RAISES THE QUESTION – WHAT ARE THE MAXIMUM POSSIBLE PRACTICAL GAINS?

Stationary Frame PI Regulator Maximum Gains

- Forward path transfer function with regulator delay T_d

$$H(s)e^{-sT_d}\,2V_{DC}G(s)=\frac{K_p 2V_{DC}}{R}\left(\frac{1+s\tau_i}{s\tau_i}\right)e^{-sT_d}\left(\frac{1}{1+sT_p}\right)$$

- Transfer function phase angle at cross over frequency $s = j\omega_c$

$$\angle\{H(j\omega_c)2V_{DC}G(j\omega_c)\}=\tan^{-1}(\omega_c\tau_i)-\pi/2-\omega_c T_d-\tan^{-1}(\omega_c T_p)=-\pi+\phi_m$$

$$\omega_c >> \frac{1}{T_p} \Rightarrow \tan^{-1}(\omega_c T_p)\approx \pi/2 \Rightarrow \omega_c=\frac{\tan^{-1}(\omega_c\tau_i)-\phi_m}{T_d}\approx\frac{\pi/2-\phi_m}{T_d}$$

Hence for a given regulator delay, ω_c is the system bandwidth at ϕ_m = selected phase margin

Stationary Frame PI Regulator Maximum Gains

- Maximum possible K_p is for a given ω_c is:

$$K_p = \frac{R}{2V_{DC}}\omega_c\tau_i\sqrt{\frac{\left(1+\omega_c{}^2T_p{}^2\right)}{\left(1+\omega_c{}^2\tau_i{}^2\right)}} \Rightarrow K_p \approx \frac{\omega_c L}{2V_{DC}}$$

assuming $\omega_c^2 T_p^2 >> 1$ and $\omega_c^2 \tau_i^2 >> 1$

- Integral time constant τ_i can be determined by making

$$\tan^{-1}\left(\omega_c\tau_i\right) \approx \pi/2 \Rightarrow \tau_i \approx \frac{10}{\omega_c}$$

- These gains are independent of the plant time constant, and are determined only by ϕ_m T_d V_{DC} and L

For any given system with regulator delay T_d , these are the maximum possible PI gains for a phase margin of ϕ_m

Example Grid Inverter: Calculations & Results

Circuit Parameter		Value
Resistive load	(R) (Ω)	1.2
Inductive load	(*L*) (mH)	10.0
Switching Freq.	(f_s) (kHz)	5.0
DC Bus volt.	($2V_{DC}$) (V)	400
Back EMF volt.	(V_{EMF}) (V_{RMS})	220
Back EMF freq.	(Hz)	50
Sampling period	(T) (sec)	10^{-4}

Circuit parameters

$$T_d = 0.75/5e^3 = 1.5e^{-4}\ \text{sec}$$

$$\phi_m = 40^o = 0.698\ \text{rad}$$

$$\omega_c = \frac{\pi/2 - \phi_m}{T_d} = \frac{1.56 - 0.698}{1.5e^{-4}} = 5.81\text{krad/s}$$

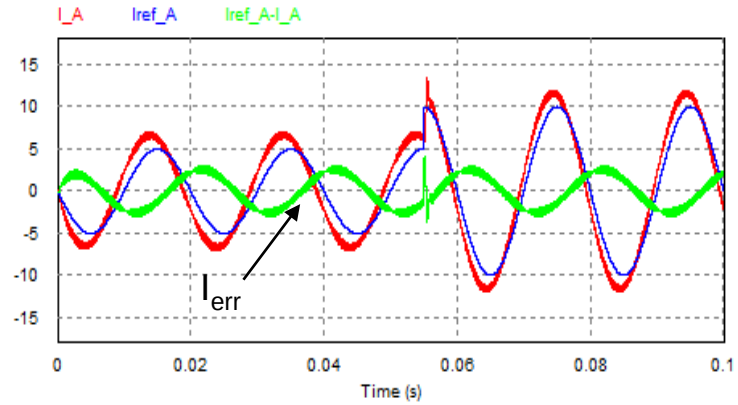
$$K_p = \frac{\omega_c L}{2V_{DC}} = \frac{5.81e^3 * 10e^{-3}}{400} = 0.145\ \text{V/A}$$

$$\tau_i = \frac{10}{\omega_c} = \frac{10}{5.82e^3} = 1.72\ \text{msec}$$

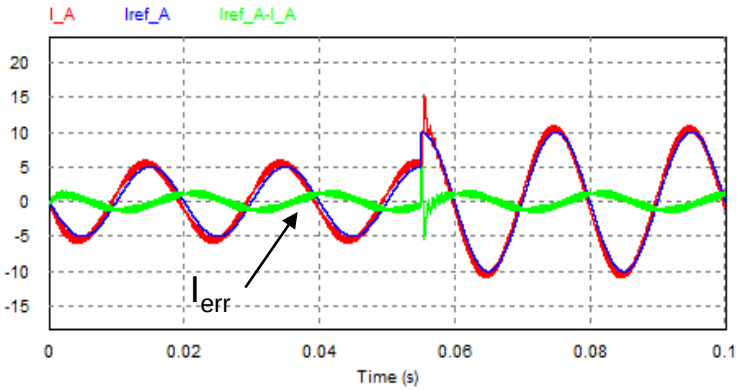
Practical Linear AC Current Regulated System

Circuit Parameter	Value
Resistive load (R) (Ω)	1.2
Inductive load (*L*) (mH)	10
Switching Freq. (f_s) (kHz)	5.0
DC Bus volt. ($2V_{DC}$) (V)	400
Back EMF volt. (V_{EMF}) (V_{RMS})	220
Back EMF freq. (Hz)	50
Sampling period (T) (sec)	10^{-4}

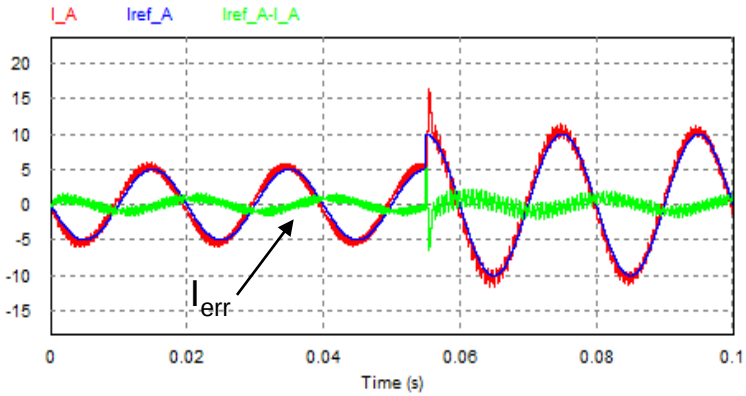
- Damping response of transient depends on designed gain margin.
- $\phi = 40^o$ is a typical value that gives a good balanced response.
- But are these gains enough to get an acceptable AC regulator performance?



$K_p = 0.145$
$\phi = 40^o$
Steady state error, damped transient



$K_p = 0.218$
$\phi = 15^o$
Reducing steady state error, underdamped transient



$K_p = 0.247$
$\phi = 5^o$
Less steady state error, very underdamped transient

Practical PI Regulator Performance Evaluation

- The current error ΔI(s) as a function of control inputs is :

$$\Delta I(s) = I_{ref}(s)\frac{1}{1+H(s)e^{-sT_d}\,2V_{DC}G(s)} + E_o(s)\frac{G(s)}{1+H(s)e^{-sT_d}\,2V_{DC}G(s)} = \Delta I_I(s) + \Delta I_D(s)$$

- It has two elements: Closed Loop Tracking Error - the ability of regulation system to follow current reference:

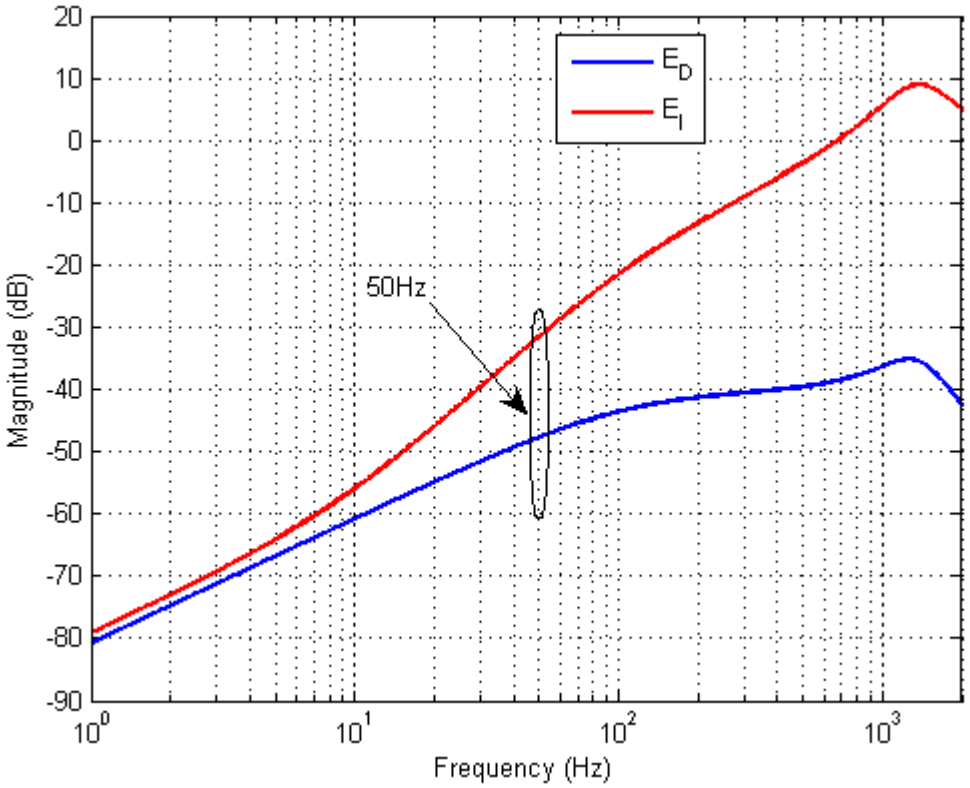
$$E_I(s) = \frac{\Delta I_I(s)}{I_{ref}(s)} = \frac{1}{1+H(s)e^{-sT_d}\,2V_{DC}G(s)}$$

- Closed Loop Disturbance Rejection Error – regulation system sensitivity to backemf disturbance:

$$E_D(s) = \frac{\Delta I_D(s)}{E_o(s)} = \frac{G(s)}{1+H(s)e^{-sT_d}\,2V_{DC}G(s)}$$

Practical PI Regulator Performance Evaluation

- Both tracking error and disturbance rejection error increase with frequency (controller gain reduces)
- Disturbance error increases more slowly because of plant pole
- BUT, magnitude of disturbance injection (emf) is usually larger
  → still creates more error



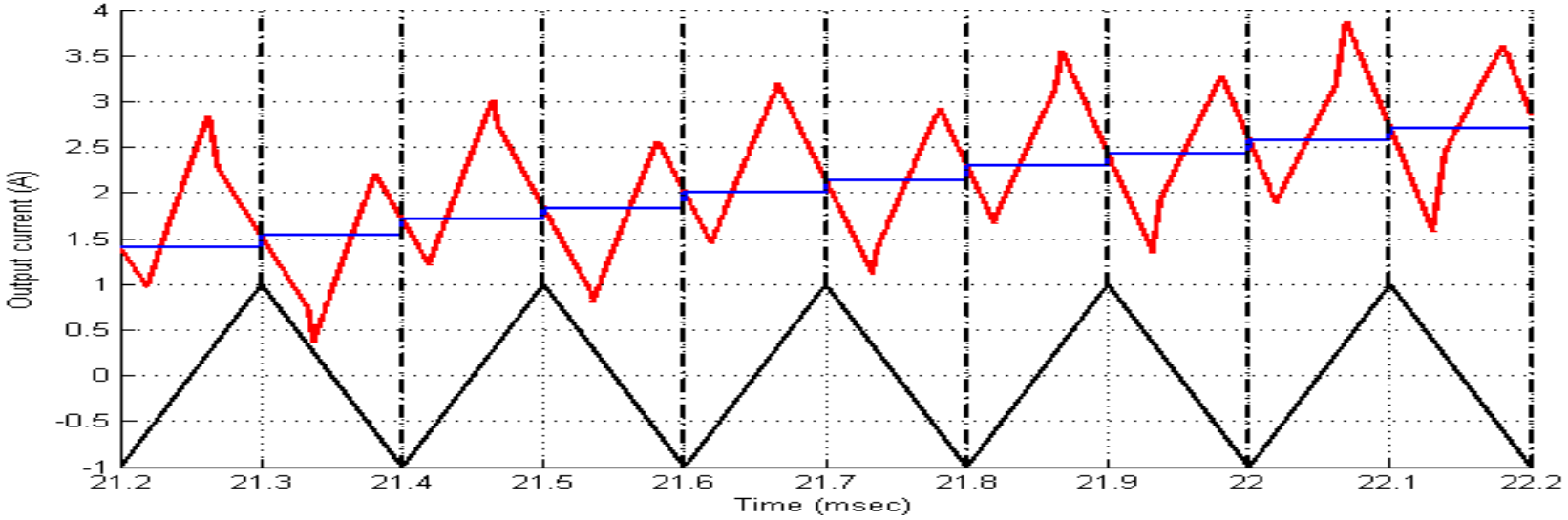
Irrespective of the contribution, the error at AC fundamental frequency is often unacceptably large with PI controller even with maximum possible gains.

Stationary Frame Regulator Enhancements

- Maximised gains are often insufficient to achieve acceptable regulator performance, especially when error caused by back EMF is considered
- The following enhancements can be considered
  - Minimise regulator delay to achieve the maximum possible gains. Use asymmetrical regulator sampled PWM and synchronous sampling as a matter of course.
  - Reduce the back EMF disturbance effect with feed forward compensation
  - Increase the regulator gain at the fundamental target frequency (Proportional-Resonant Controller)
- Lets see how these ideas work

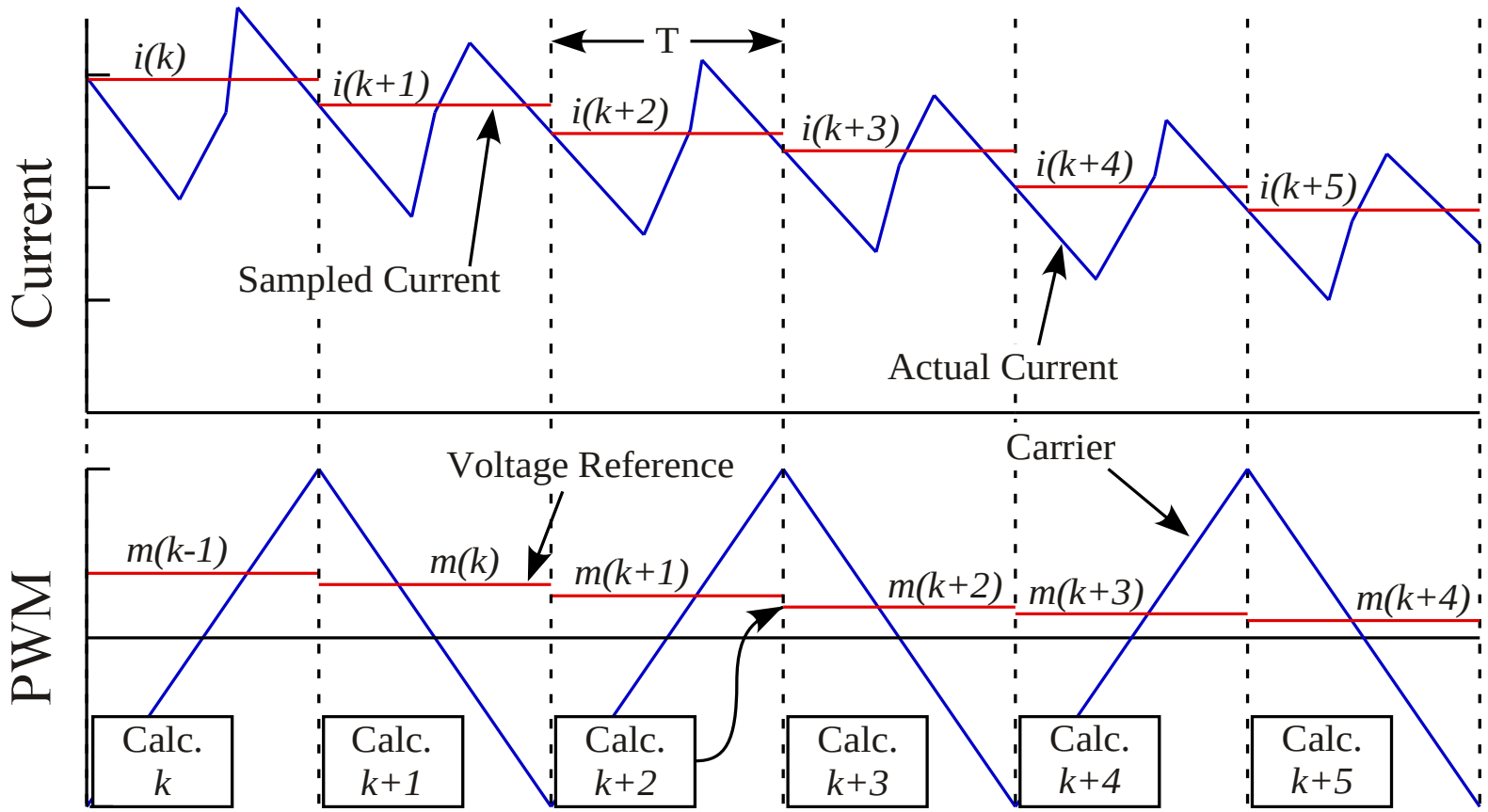
Synchronously sampling the “average” current

- Require to measure average load current (no switching harmonic components) without filter delay, to avoid increasing T_d.
- Synchronously sample at top/bottom of (implied) carrier
- At this point VSI phase leg volt-second output equals commanded voltage → minimum measured current ripple



Synchronous sampling of measured current

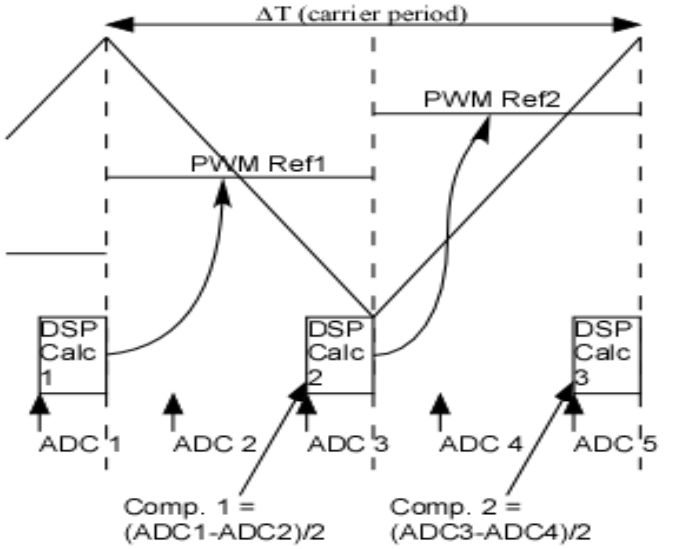
Synchronously sampling the “average” current

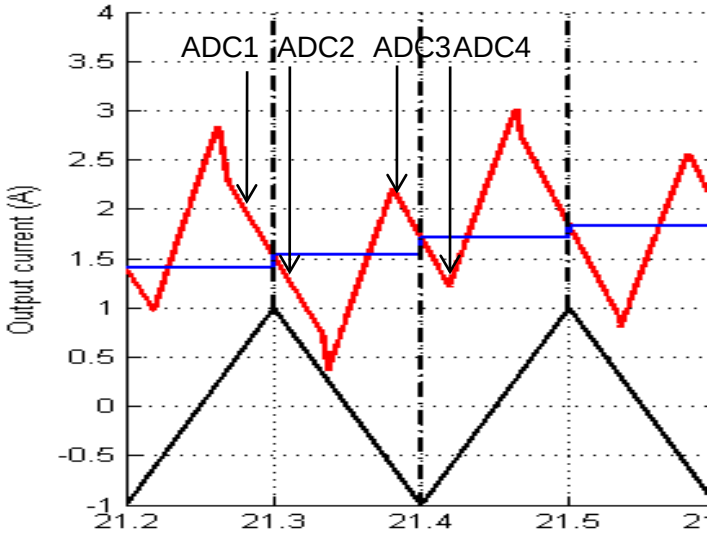


- Ensures that the sampled currents correspond to the average currents
- Still has ½ carrier interval transport delay due to finite calculation time

Further Minimise Transport Delay

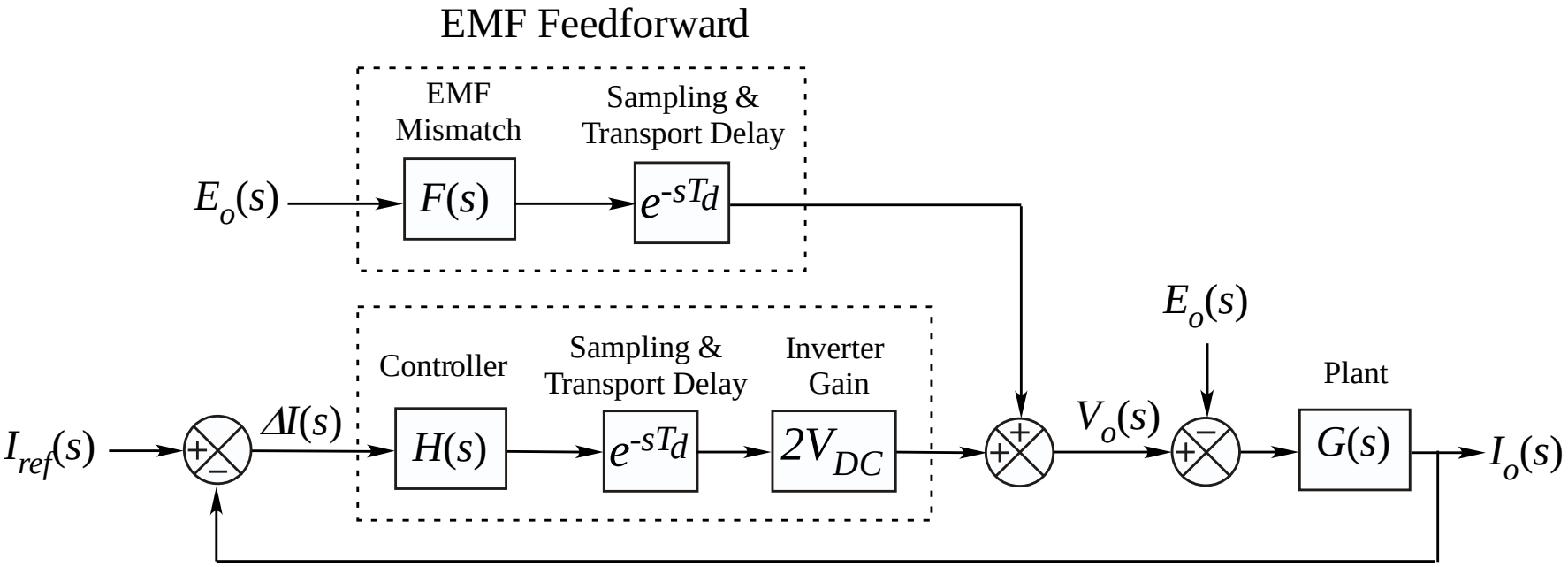
- At low switching frequencies, tracking error can still be too high
- Further reducing transport delay allows higher gains
- Achieved by pre sampling before each half carrier cycle and dynamically compensating for ripple current in sample
- Feasible using modern DSP with synchronous sampling





Feedforward Mitigation of Backemf Disturbance

- Back EMF is predictable, and so its effect can be reduced by feedforwarding the measured (or estimated) value



Average model AC current regulation system with back EMF feedforward

Feedforward Mitigation of Backemf Disturbance

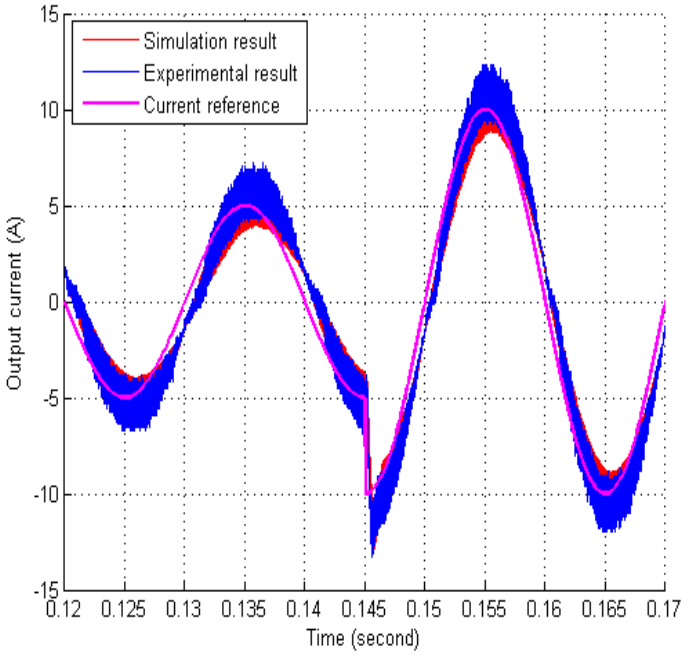
- The transfer function of system with feedforward becomes:

$$I_o(s) = I_{ref}(s)\frac{H(s)e^{-sT_d}\,2V_{DC}G(s)}{1+H(s)e^{-sT_d}\,2V_{DC}G(s)} - E_o(s)\frac{\left(1-F(s)e^{-sT_d}\right)G(s)}{1+H(s)e^{-sT_d}\,2V_{DC}G(s)}$$

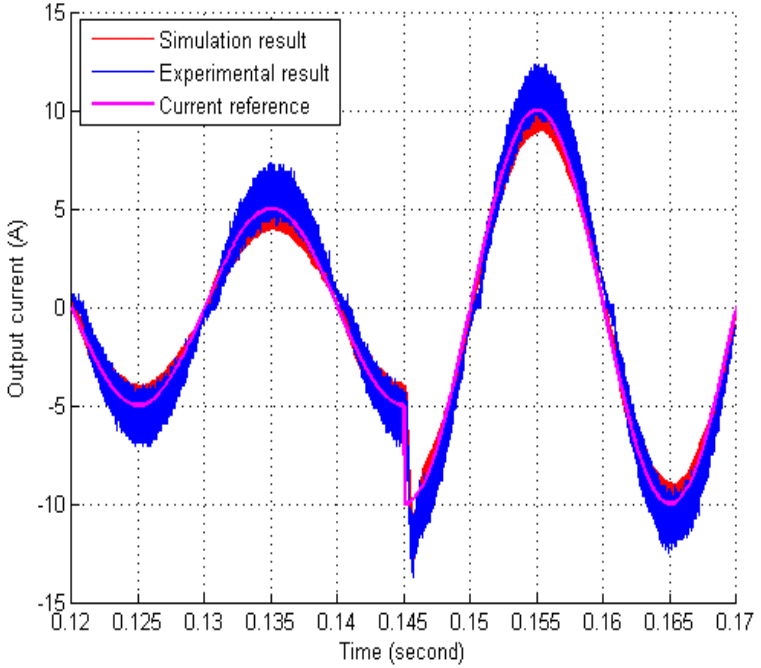
$$\Delta I(s) = I_{ref}(s)\frac{1}{1+H(s)e^{-sT_d}\,2V_{DC}G(s)} + E_o(s)\frac{\left(1-F(s)e^{-sT_d}\right)G(s)}{1+H(s)e^{-sT_d}\,2V_{DC}G(s)} = \Delta I_I(s) + \Delta I_D(s)$$

- With perfect feedforward (F(s)=1) influence of backemf is completely eliminated ($E_D(s)=0$), and only $E_R(s)$ remain.
- The controller design process still maximises K_P as before.

Feedforward Mitigation of Backemf Disturbance



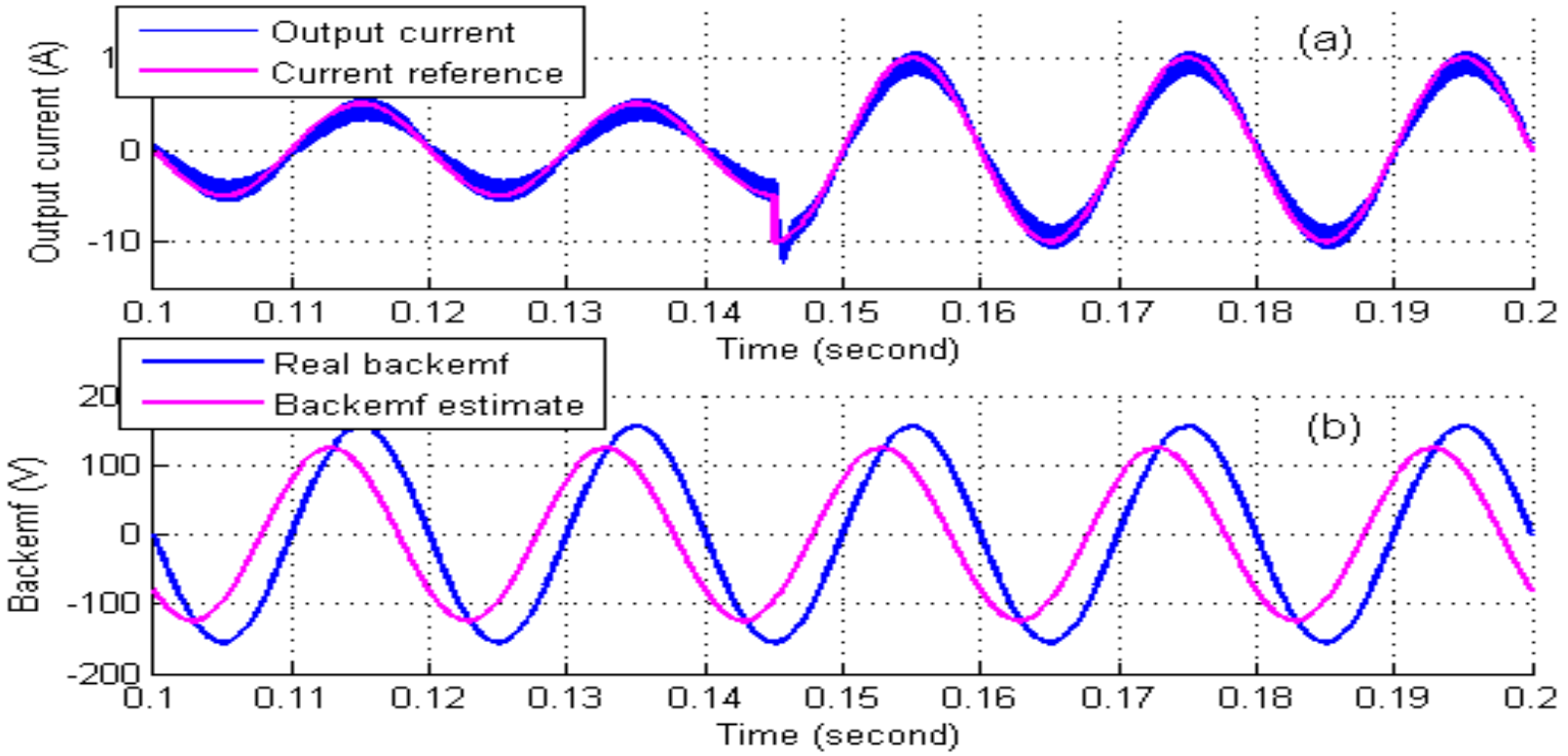
PI controller without backemf feed forward



PI controller with backemf feed forward

Simulated and Experimental system response with 110Vrms backemf.

Accuracy of Back EMF Estimation



(a) Performance of PI regulator with backemf feedforward

(b) Miss estimated back EMF estimation (80% magnitude, 40^{o} phase lead)

Increase Fundamental Gain with PR Regulator

- Key limitation of PI regulator is high $E_D(s)$ at fundamental reference frequency (ω_0)
- An alternative approach is to increase controller gain at ω_0 without affecting high frequency gain determined by K_p
- This is the strategy of the P+Resonant (PR) controller, with a resonant gain peak at ω_0 :

$$H_{PR}(s) = K_p\left(1 + \frac{1}{\tau_i}\frac{s}{s^2 + \omega_o{}^2}\right)$$

Increase Fundamental Gain with PR Regulator

- Properties of the P+Resonant controller:
  - Low and high frequency behaviour approximates a proportional controller
  - Controller provides near infinite gain at the target AC frequency
  - Reference tracking error and disturbance error are near zero at the AC frequency
  - Hence zero steady magnitude and phase error

Note : PR regulator can be a little sensitive to mismatch between the resonant term and AC fundamental frequency.

Maximum Gains with PR Regulator

- Forward Path Gain:

$$H_{PR}(s)e^{-sT_d}\,2V_{DC}G(s)=\frac{K_p 2V_{DC}}{R}\left(1+\frac{1}{\tau_i}\frac{s}{s^2+\omega_o^2}\right)e^{-sT_d}\left(\frac{1}{1+sT_p}\right)$$

$$T_p=\frac{L}{R}$$

- At the cross-over frequency: $s=j\omega_c$

$$H_{PR}(j\omega_c)e^{-j\omega_c T_d}\,2V_{DC}G(j\omega_c)=\frac{K_p 2V_{DC}}{R}\left(1+\frac{1}{\tau_i}\frac{j\omega_c}{\omega_o^2-\omega_c^2}\right)e^{-j\omega_c T_d}\left(\frac{1}{1+j\omega_c T_p}\right)$$

- If $\omega_c >> \omega_o$, then:

$$FORWARD\ PATH\ GAIN \approx \frac{K_p 2V_{DC}}{R}\left(1+\frac{1}{j\omega_c\tau_i}\right)e^{-j\omega_c T_d}\left(\frac{1}{1+j\omega_c T_p}\right)$$

$$=\frac{K_p 2V_{DC}}{R}\left(\frac{1+j\omega_c\tau_i}{j\omega_c\tau_i}\right)e^{-j\omega_c T_d}\left(\frac{1}{1+j\omega_c T_p}\right)$$

- This is identical to the PI expression.

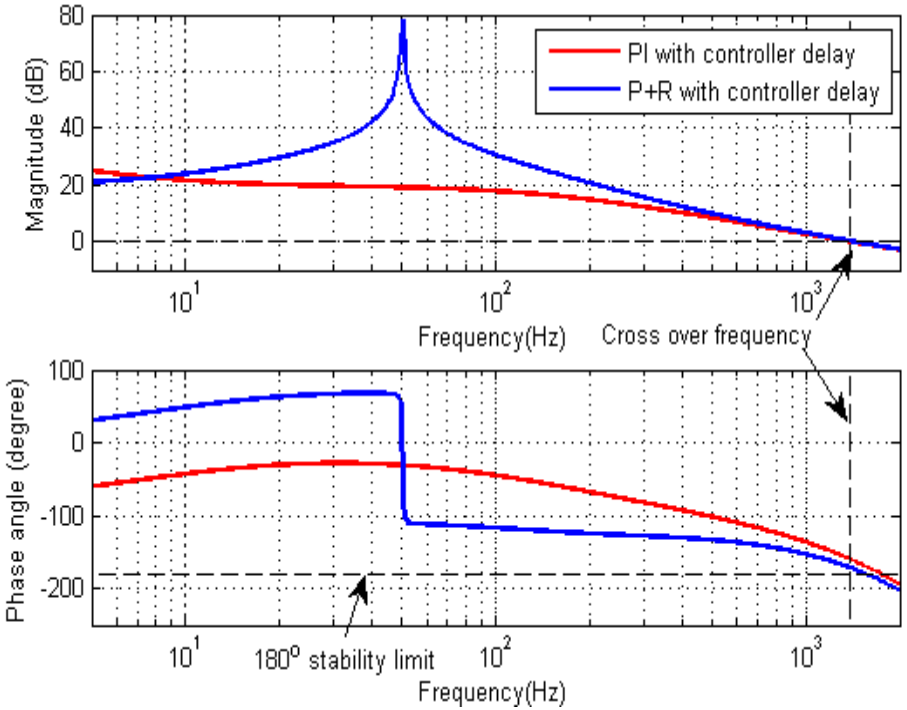
$$H(s)e^{-sT_d}\,2V_{DC}G(s)=\frac{K_p 2V_{DC}}{R}\left(\frac{1+s\tau_i}{s\tau_i}\right)e^{-sT_d}\left(\frac{1}{1+sT_p}\right)$$

Maximum Gains with PR Regulator

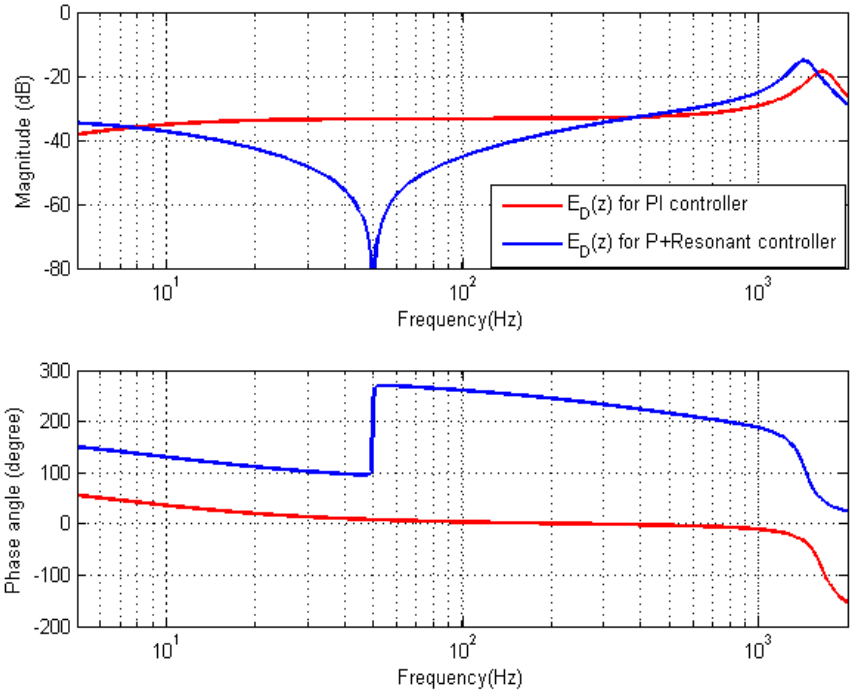
- Therefore the maximum gains for the PR controller are the same as for the PI controller, and are defined by:
  - Target Phase Margin ϕ_m
  - DC link voltage $2V_{DC}$
  - Load Inductance L
  - PWM/Current regulator transport delay T_d

$$\omega_c = \frac{\pi/2 - \phi_m}{T_d} \qquad K_p = \frac{\omega_c L}{2V_{DC}} \qquad \tau_i = \frac{10}{\omega_c}$$

Maximum Gains with PR Regulator



Bode plot of GH(s) for PI and PR regulators with transport delays

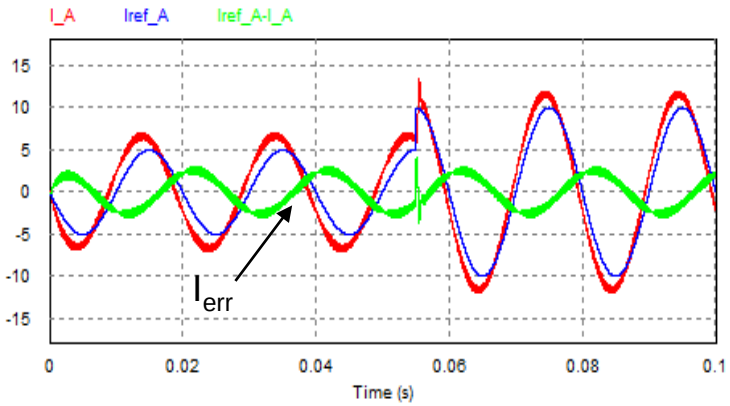


Closed Loop Disturbance Rejection Error $E_D(s)$ for PI and PR controllers

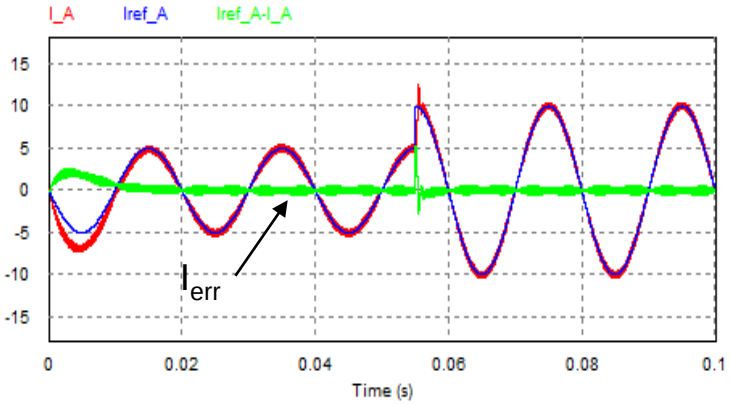
- Resonant gain peak improves backemf disturbance rejection at ω_0
- Also reduce $E_I(s)$ at ω_0, leading to better tracking

Practical Linear AC Current Regulated System

Circuit Parameter		Value
Resistive load	(R) (Ω)	1.2
Inductive load	(*L*) (mH)	10
Switching Freq.	(f_s) (kHz)	5.0
DC Bus volt.	($2V_{DC}$) (V)	400
Back EMF volt.	(V_{EMF}) (V_{RMS})	220
Back EMF freq.	(Hz)	50
Sampling period	(T) (sec)	10^{-4}



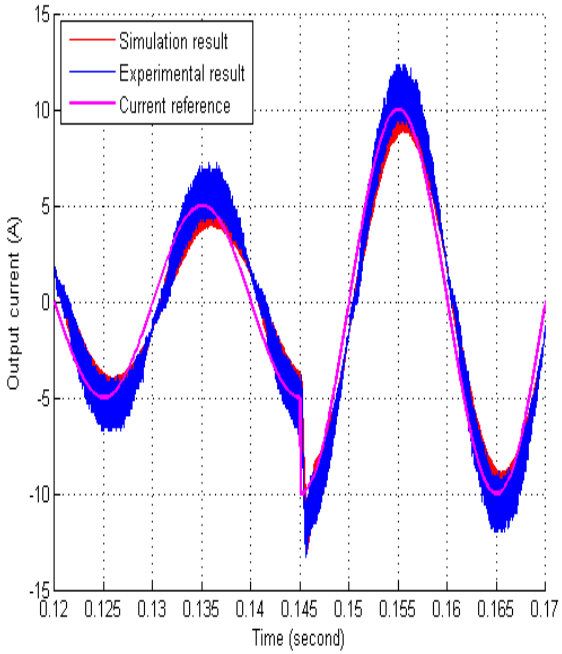
PI controller:
$K_p = 0.145$
$\phi = 40^o$
Steady state error, damped transient



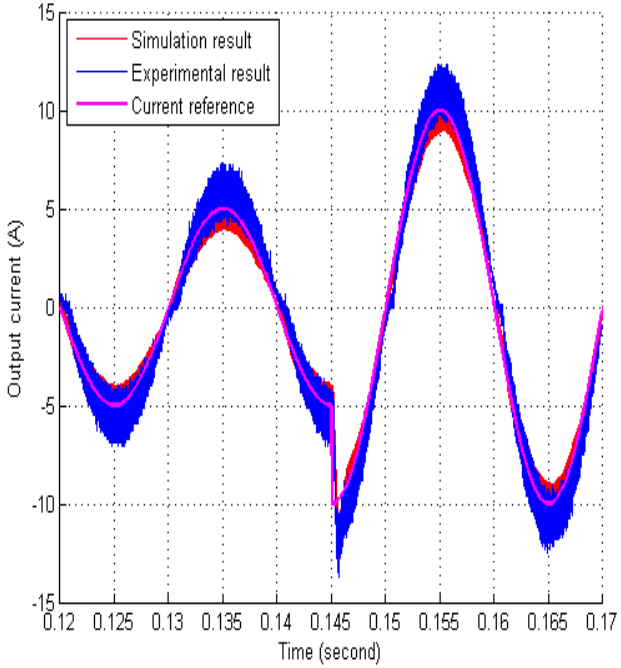
PR controller:
$K_p = 0.145$
$\phi = 40^o$
Neligible steady state error, damped transient

PI verses PR controller response with same gains.

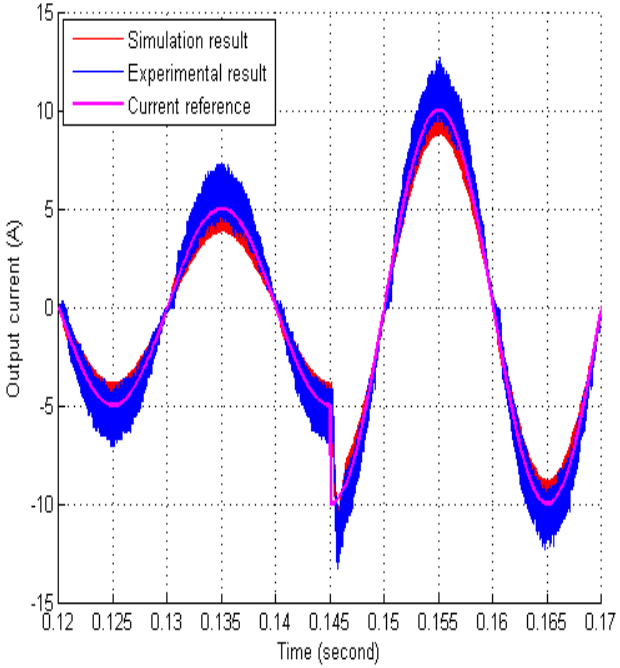
Experimental Confirmation: PR Regulator



PI controller without backemf feed forward



PI controller with backemf feed forward

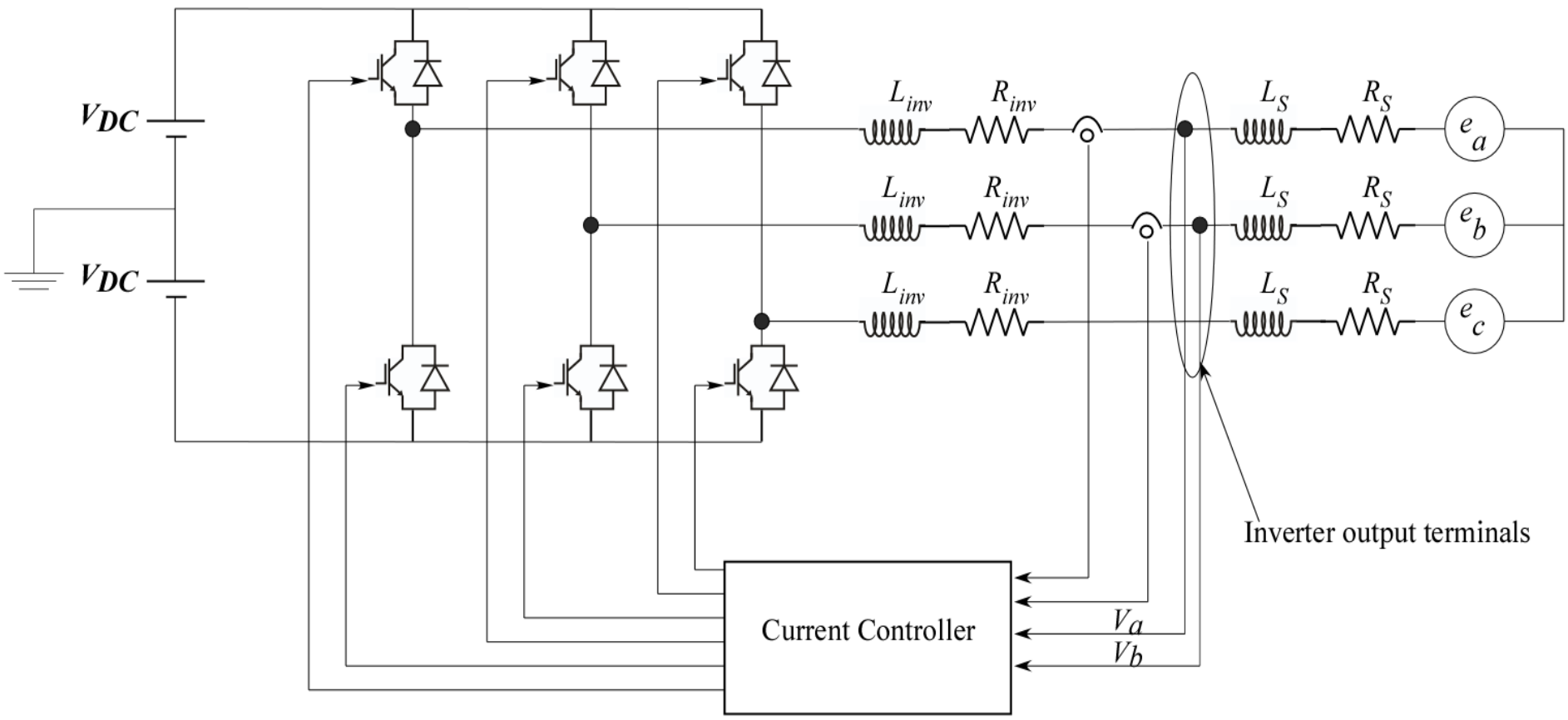


PR controller

Simulated and Experimental system response with 110Vrms backemf

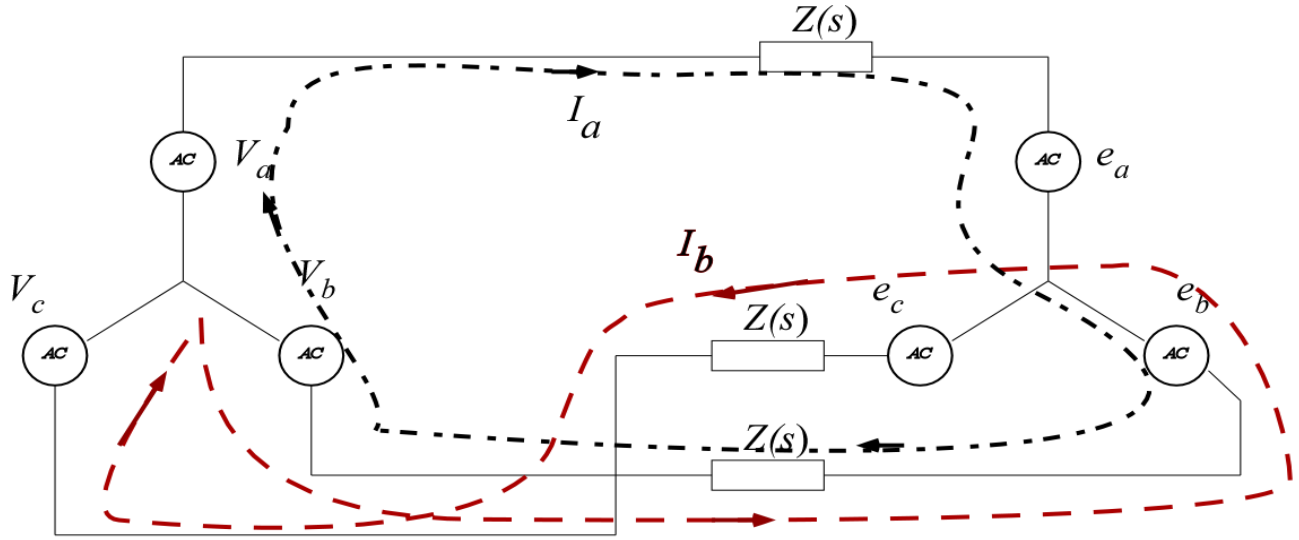
Same transient response for all three alternatives

Three Phase AC Current Regulated VSI



Three phase Voltage Source Inverter (VSI) connected to a back EMF load through a series R-L impedance

Three Phase AC Current Regulated VSI



- KVL loops:

$$V_a(s) = 2I_a(s)Z(s) - I_b(s)Z(s) + E_a(s) - E_b(s) + V_b(s)$$

$$V_b(s) = 2I_b(s)Z(s) - I_a(s)Z(s) + E_b(s) - E_c(s) + V_c(s)$$

- For balanced system:

$$V_a(s) + V_b(s) + V_c(s) = 0 \Rightarrow V_c(s) = -V_a(s) - V_b(s)$$

$$E_a(s) + E_b(s) + E_c(s) = 0 \Rightarrow E_c(s) = -E_a(s) - E_b(s)$$

Three Phase AC Current Regulated VSI

- For an isolated three phase system

$$i_a(s)+i_b(s)+i_c(s)=0$$

- Substituting voltage and current constraints into KVL loops gives

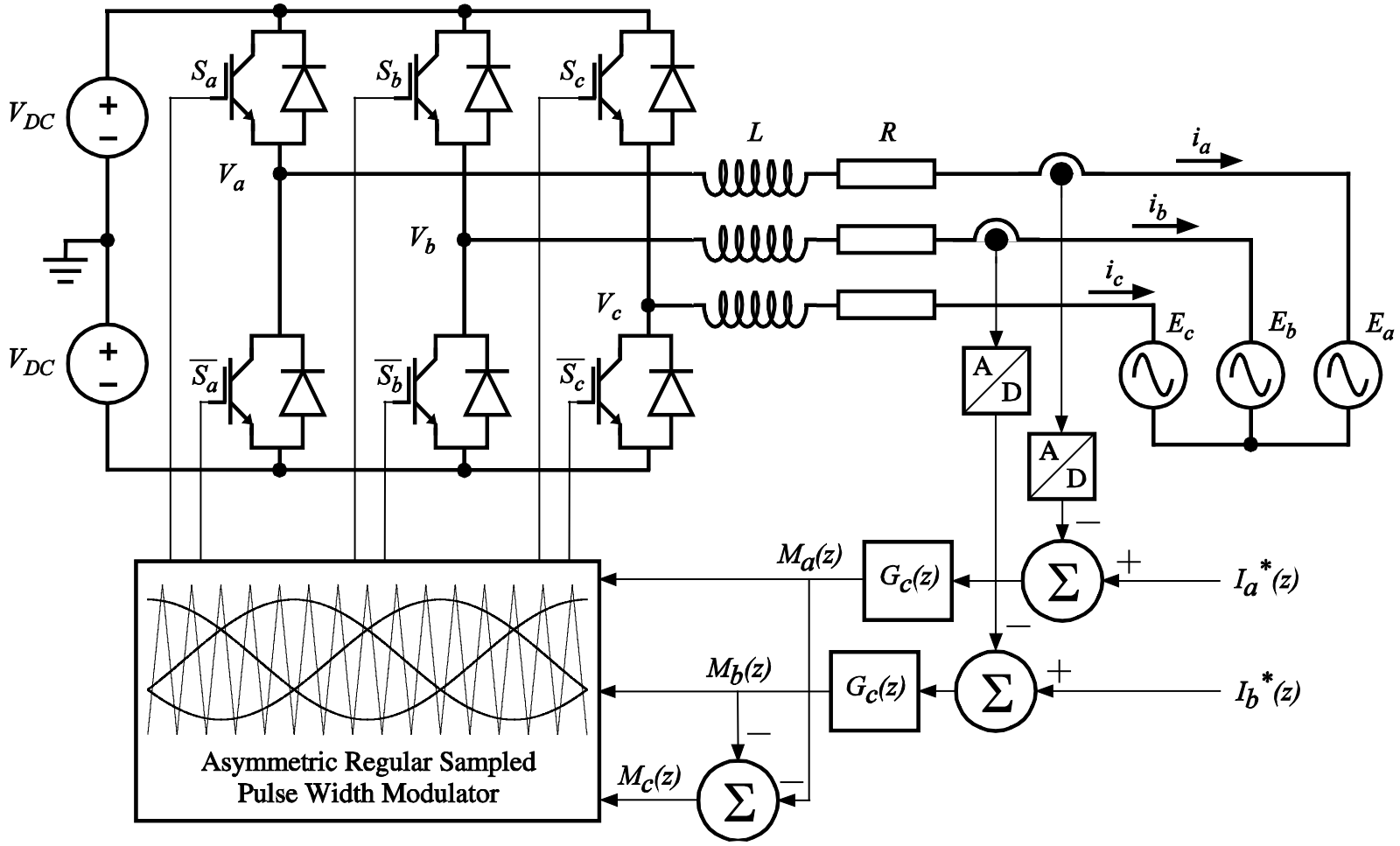
$$i_a(s)=\frac{V_a(s)-E_a(s)}{R+sL} \qquad i_b(s)=\frac{V_b(s)-E_b(s)}{R+sL} \qquad i_c(s)=\frac{V_c(s)-E_c(s)}{R+sL}$$

Thus only two currents need to be controlled for a three phase system, each regulated only by their individual phase voltages using previous single phase theory

- Modulate third phase to achieve balanced system using:

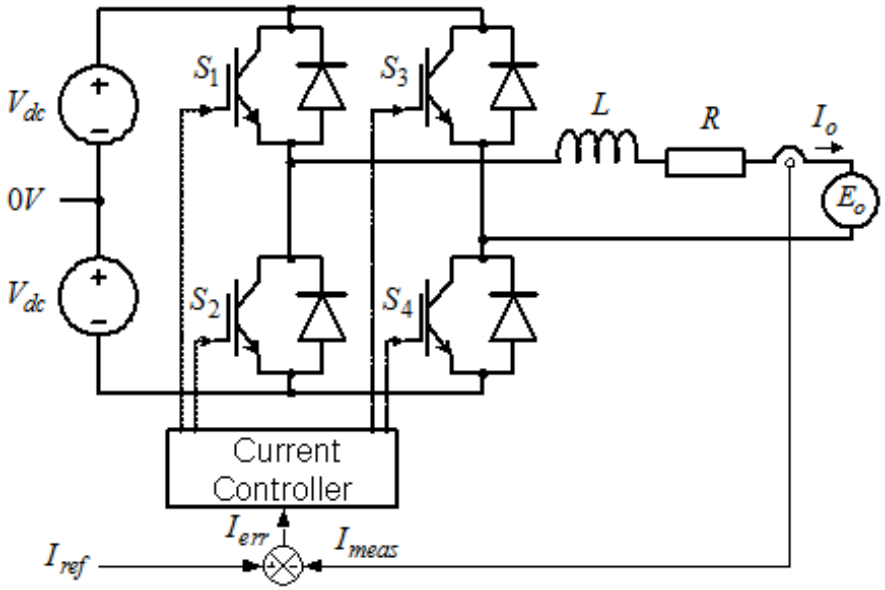
$$V_c(s)=-V_a(s)-V_b(s)$$

Three Phase AC Current Regulated VSI

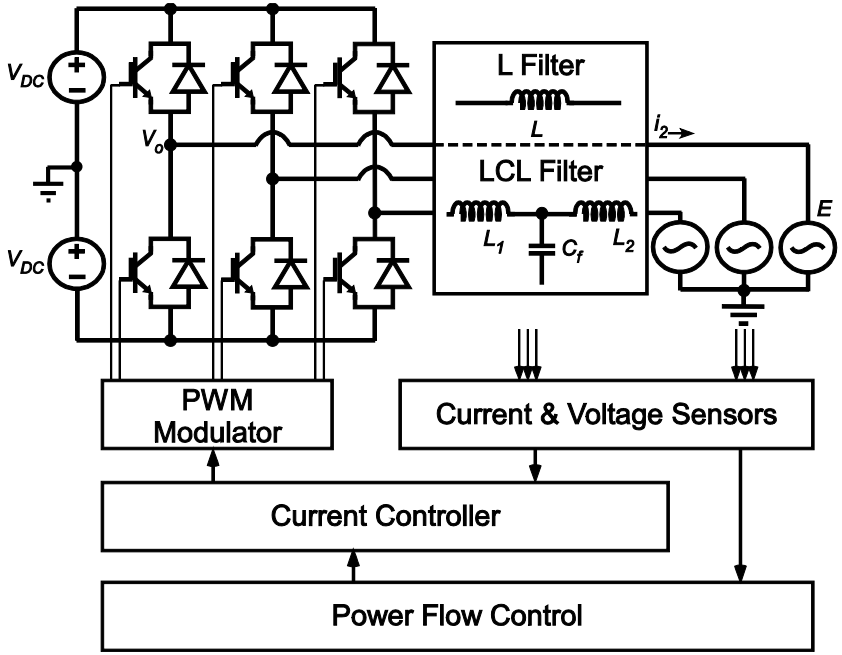


Topology of a three phase AC current regulated voltage source inverter

Three Phase AC Current Regulated VSI



Single Phase Current Regulated System



Three Phase Current Regulated System (L or LCL Filter)

Note - effective DC source voltage for K_p gain calculation changes from $2V_{DC}$ for single phase system to V_{DC} for three phase system

Example Grid Inverter: Calculations & Results

Circuit Parameter		Value
Resistive load	(R) (Ω)	1.2
Inductive load	(*L*) (mH)	20
Switching Freq.	(f_s) (kHz)	5.0
DC Bus volt.	($2V_{DC}$) (V)	400
Back EMF volt.	(V_{EMF}) (V_{RMS})	80
Back EMF freq.	(Hz)	50
Sampling period	(T) (sec)	10^{-4}

Circuit parameters

$$T_d = 0.75/5e^3 = 1.5e^{-4}\ \text{sec}$$

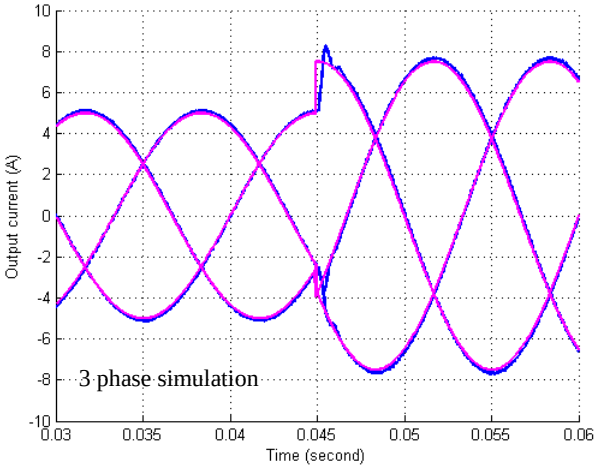
$$\phi_m = 40^o = 0.698\ \text{rad}$$

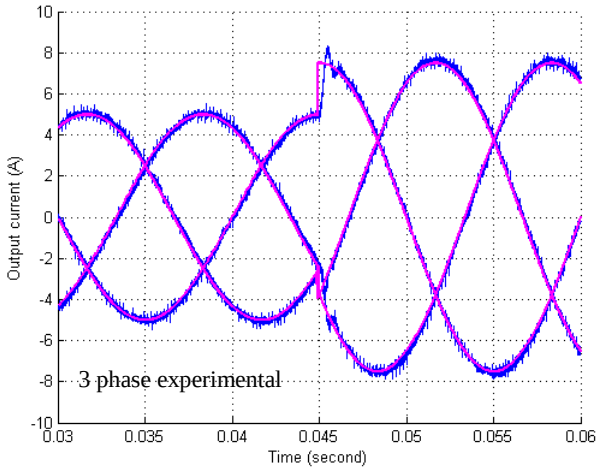
$$\omega_c = \frac{\pi/2 - \phi_m}{T_d} = \frac{1.56 - 0.698}{1.5e^{-4}} = 5.81\text{krad/s}$$

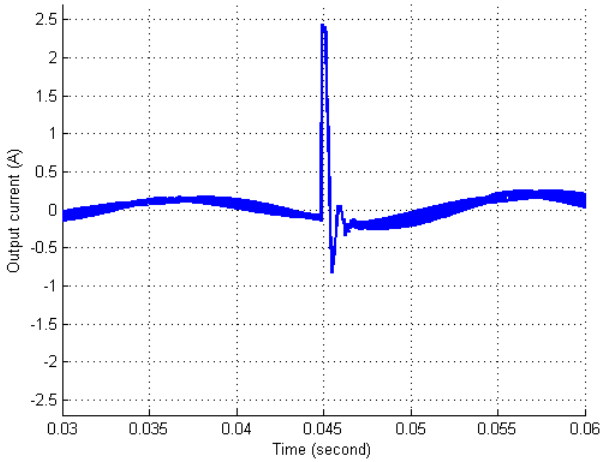
$$K_p = \frac{\omega_c L}{V_{DC}} = \frac{5.81e^3 * 20e^{-3}}{200} = 0.582\ \text{V/A}$$

$$\tau_i = \frac{10}{\omega_c} = \frac{10}{5.82e^3} = 1.73\ \text{msec}$$

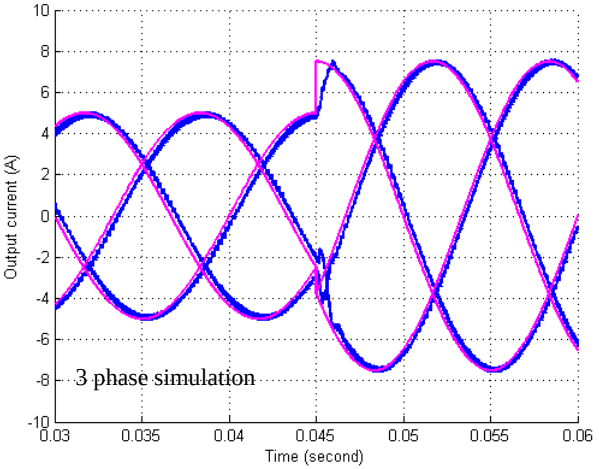
Example Grid Inverter: Calculations & Results

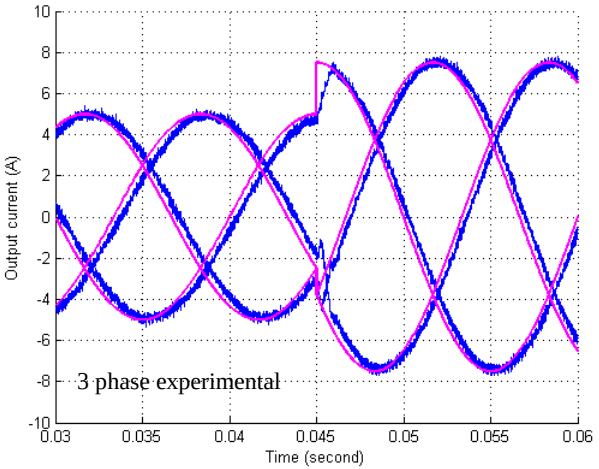


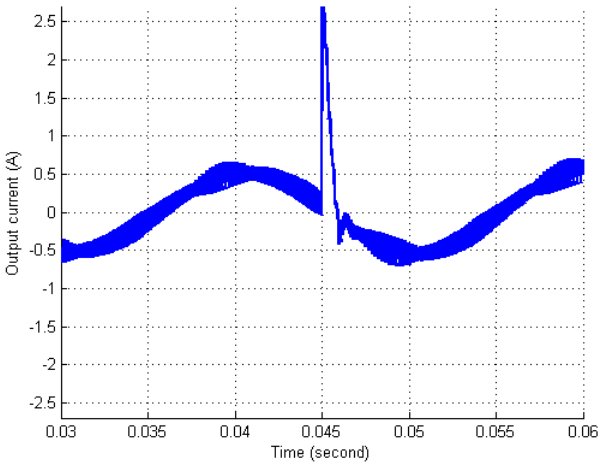




No back EMF, PI maximised gains (f_s = 5kHz)

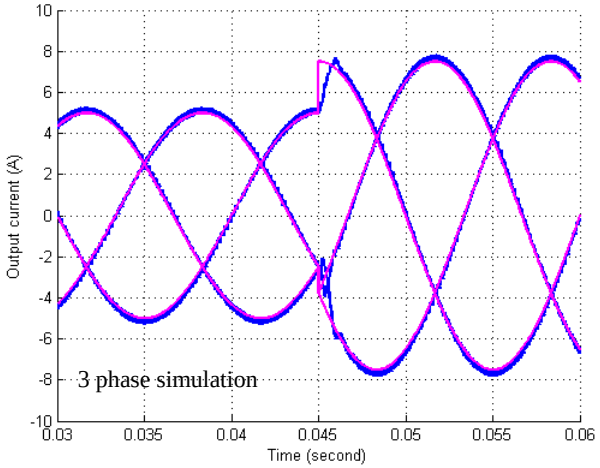


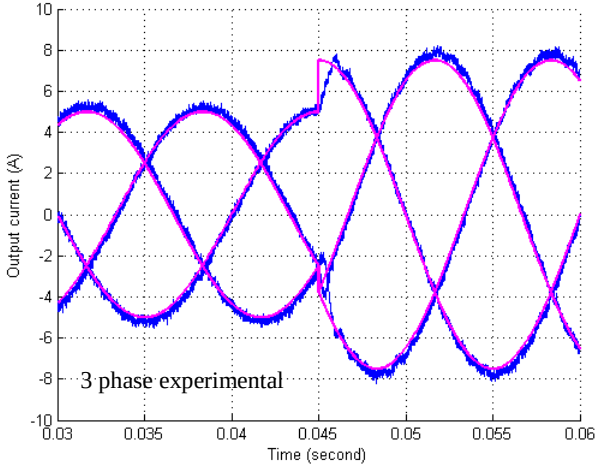


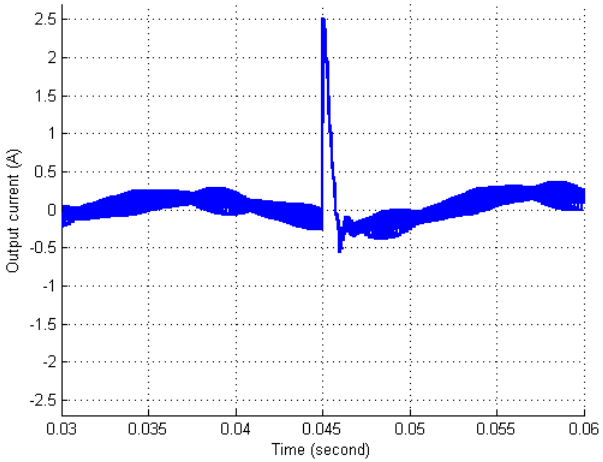


Back EMF, PI maximised gains (f_s = 5kHz)

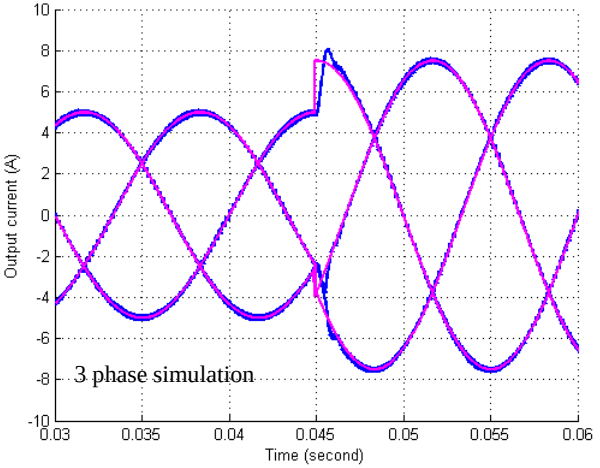
Example Grid Inverter: Calculations & Results

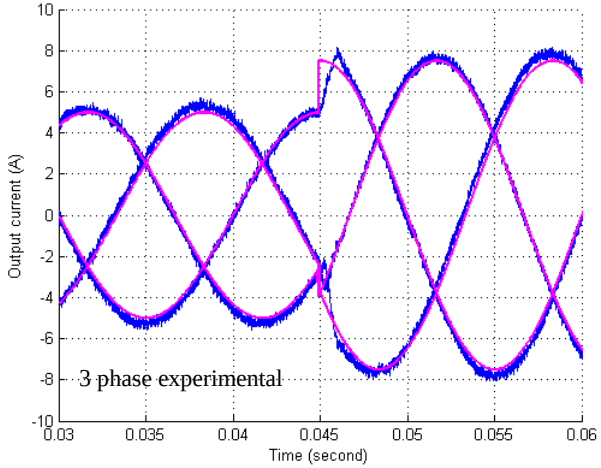


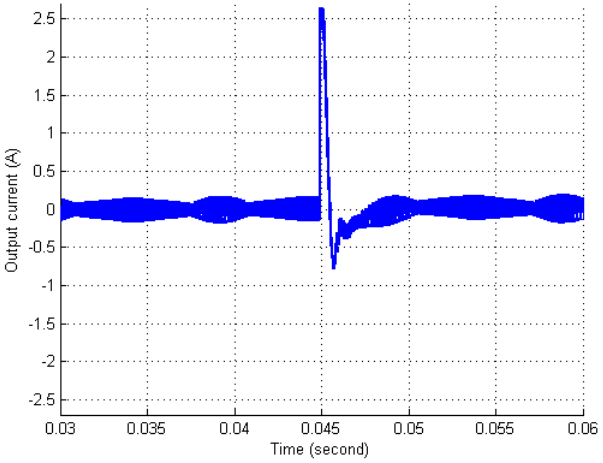




Back EMF, PI maximised gains plus feedforward EMF compensation (fs = 5kHz)

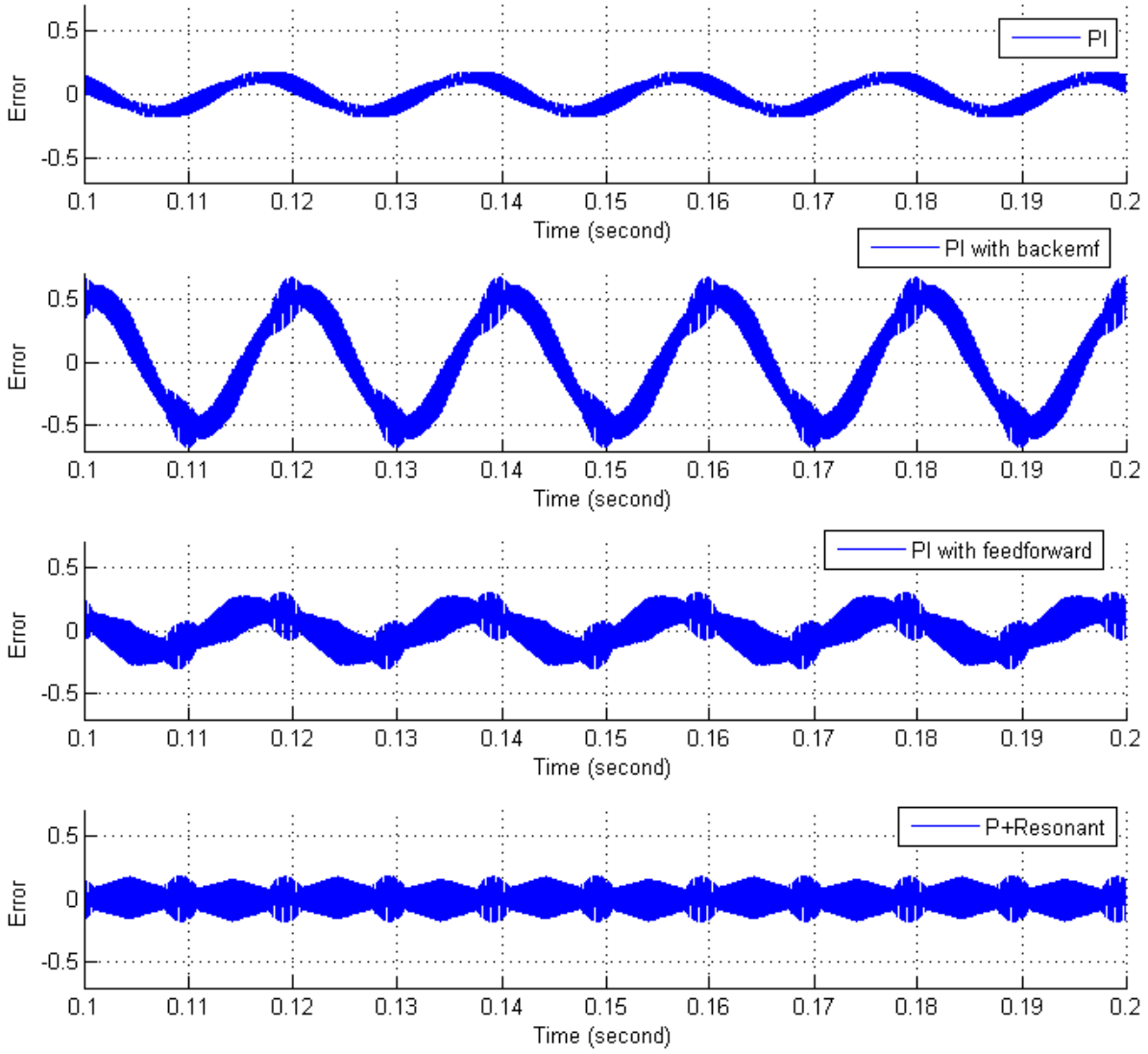






Back EMF, PR regulator maximised gains (f_s = 5kHz)

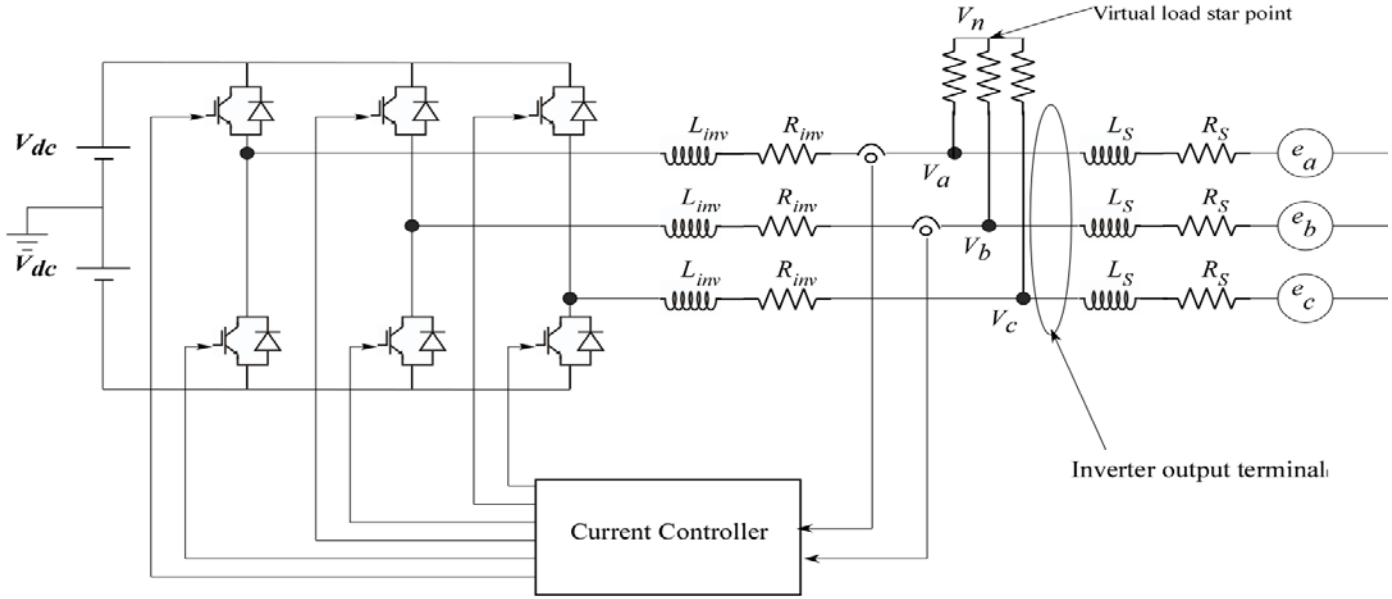
Example Grid Inverter: Calculations & Results



Comparison of Current Error – all regulator alternatives

Back EMF Estimation by Direct Measurement

- Synchronously measure scaled values of back EMF at inverter output terminals, if it has internal filter



Circuit arrangement to allow measurement of back EMF

- Back EMF given by :

$$E_{a(meas)} = \frac{Z_{filter}(s)}{Z_{filter}(s) + Z_{grid}(s)}$$

- Sensitive to noise, particularly PWM switching ripple.

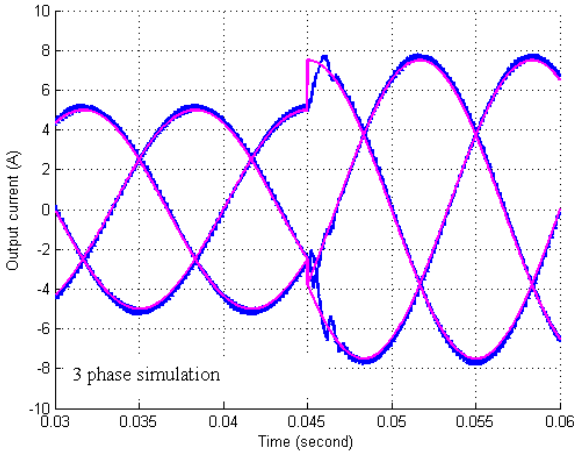
Back EMF Estimation by Predictive Calculation

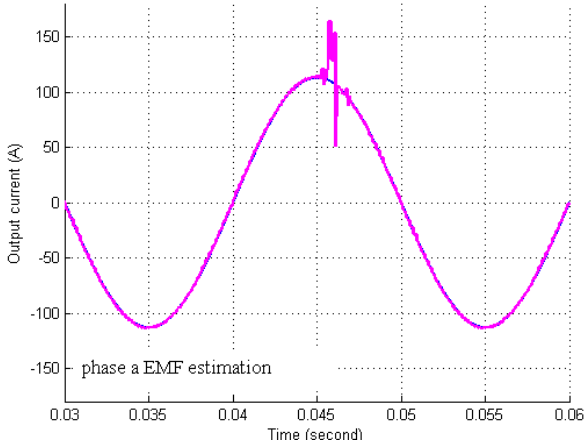
- Back EMF estimation, as used in predictive current regulator

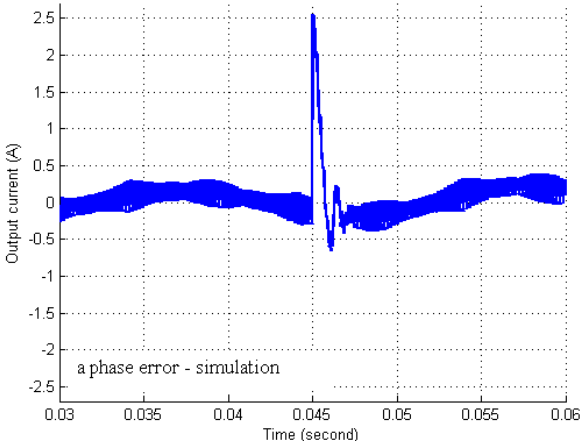
$$e_a[i] = V_a[i-2] - 2V_a[i-3] - i_a[i]R' + \frac{L'}{\Delta T/2}\left(-3i_a[i-1] + 5i_a[i-2] - 2i_a[i-3]\right)$$

- Load inductance estimation constraints

$$0.8L \le L' \le 1.23L$$



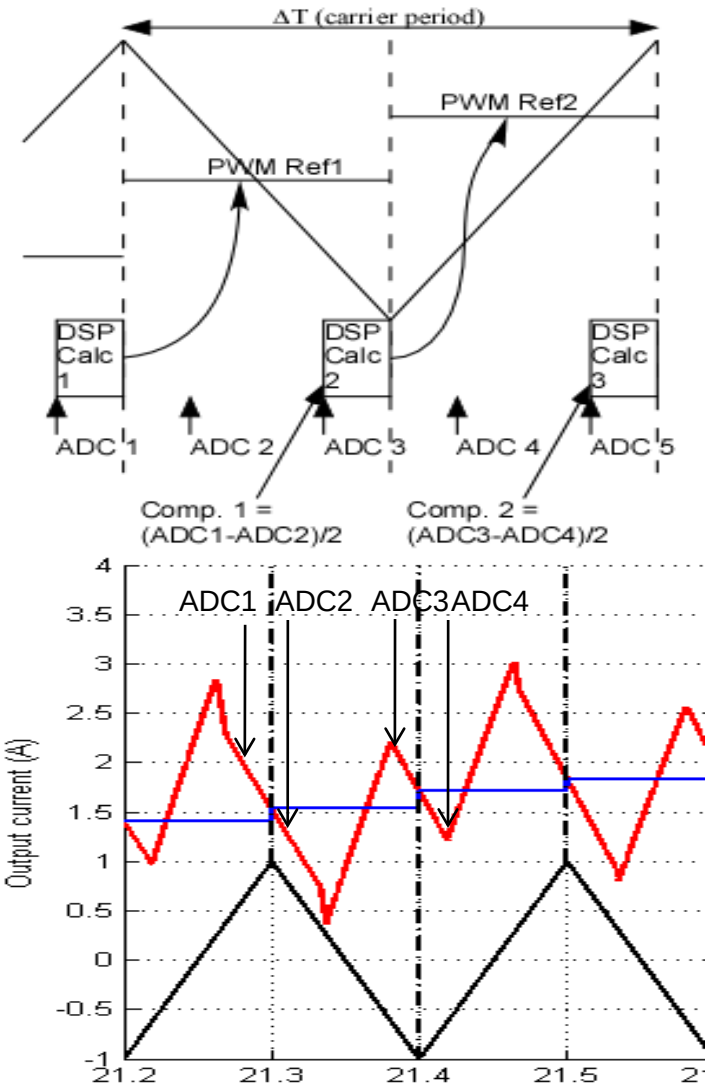




Response with back EMF, estimated feedforward EMF (f_s = 5kHz)

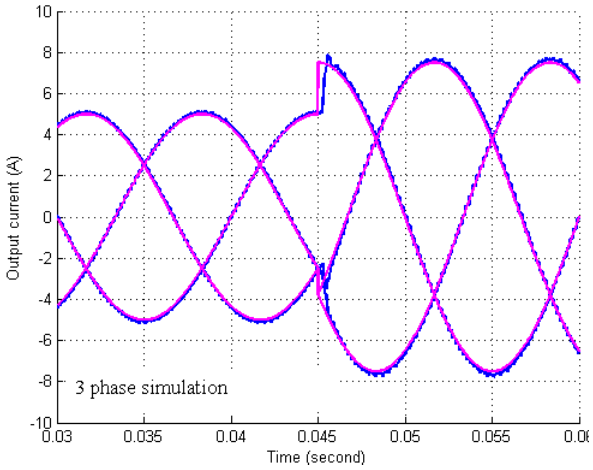
Further Minimise PWM Transport Delay

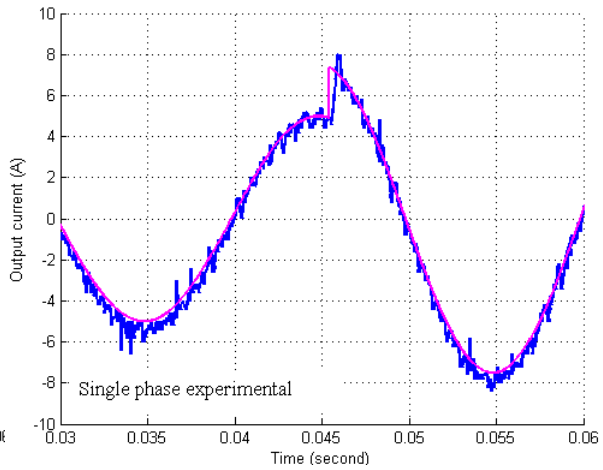
- At low switching frequencies, tracking error can still be too high
- Further reducing transport delay allows higher gains
- Achieved by pre sampling before each half carrier cycle and dynamically compensating for ripple current in sample
- Feasible using modern DSP with synchronous sampling

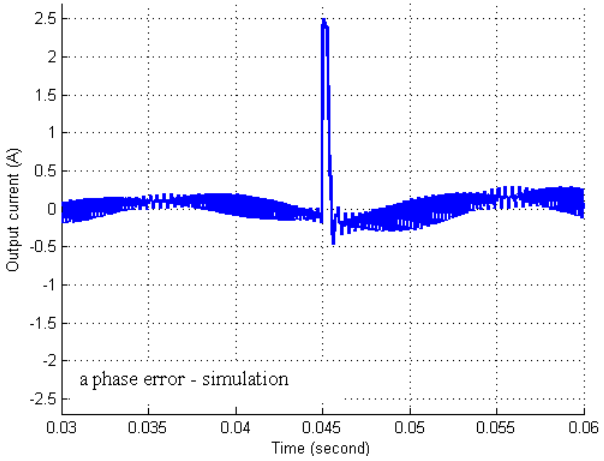


Further Minimise PWM Transport Delay

- Ideally reduce overall regulator delay to 0.25 carrier period
- Theoretically should increase ω_c and gains by factor of 3
- Experimentally found to be susceptible to noise
- Only approx 2 times increase in gain was achievable







without back EMF, PI controller, reduced sampling time by measurement compensation (**f_s = 2.5 kHz**)

Orthogonal Reference Frames: $\alpha\beta$ stationary frame

- ABC frame : Three non-orthogonal variables with two degrees of freedom
- $\alpha\beta$ frame: Two orthogonal variables
- Clarke (alpha-beta) transform:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = T_{\alpha\beta} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

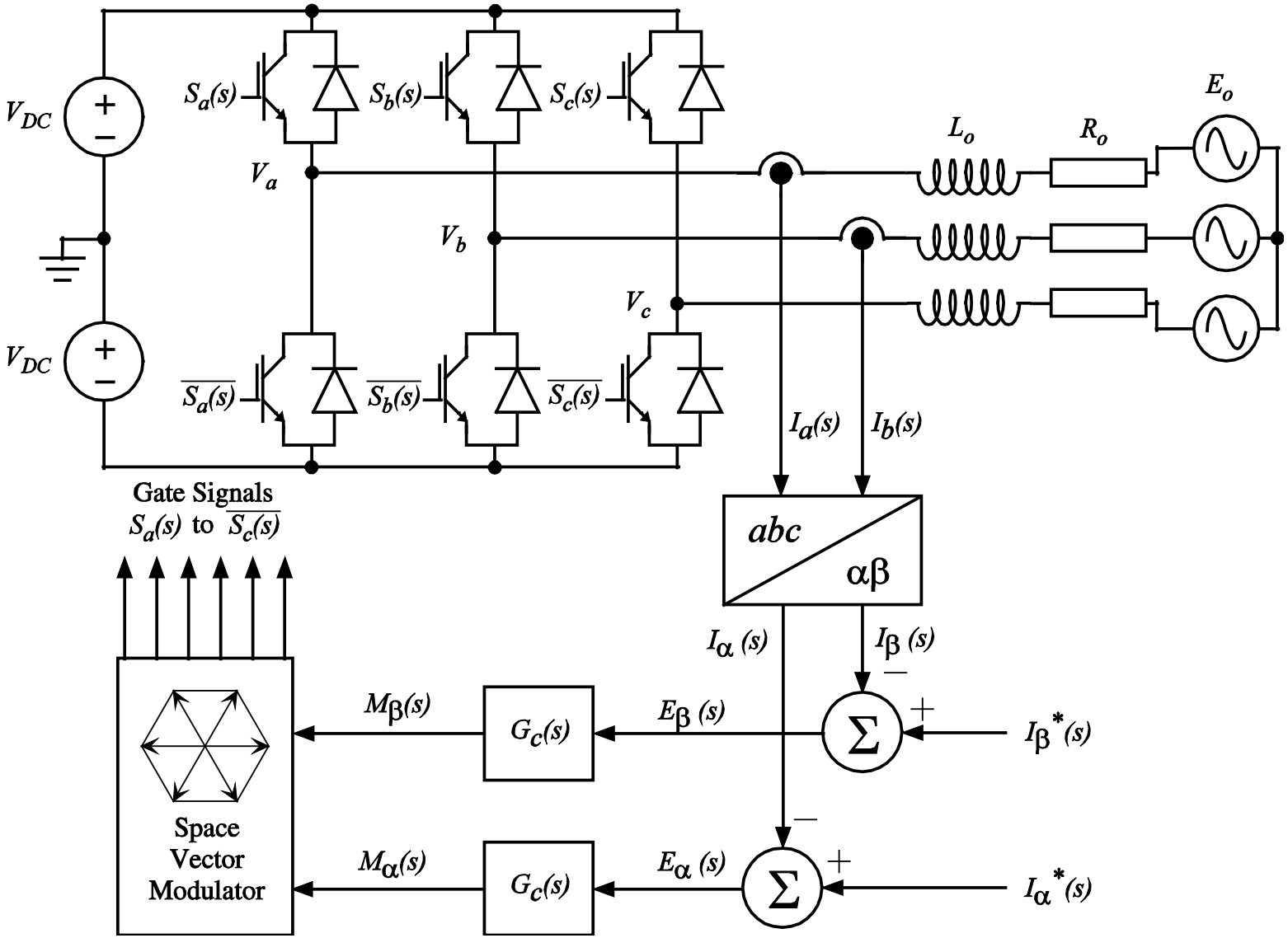
- Inverse Clarke transform:

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = T_{\alpha\beta}^T \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

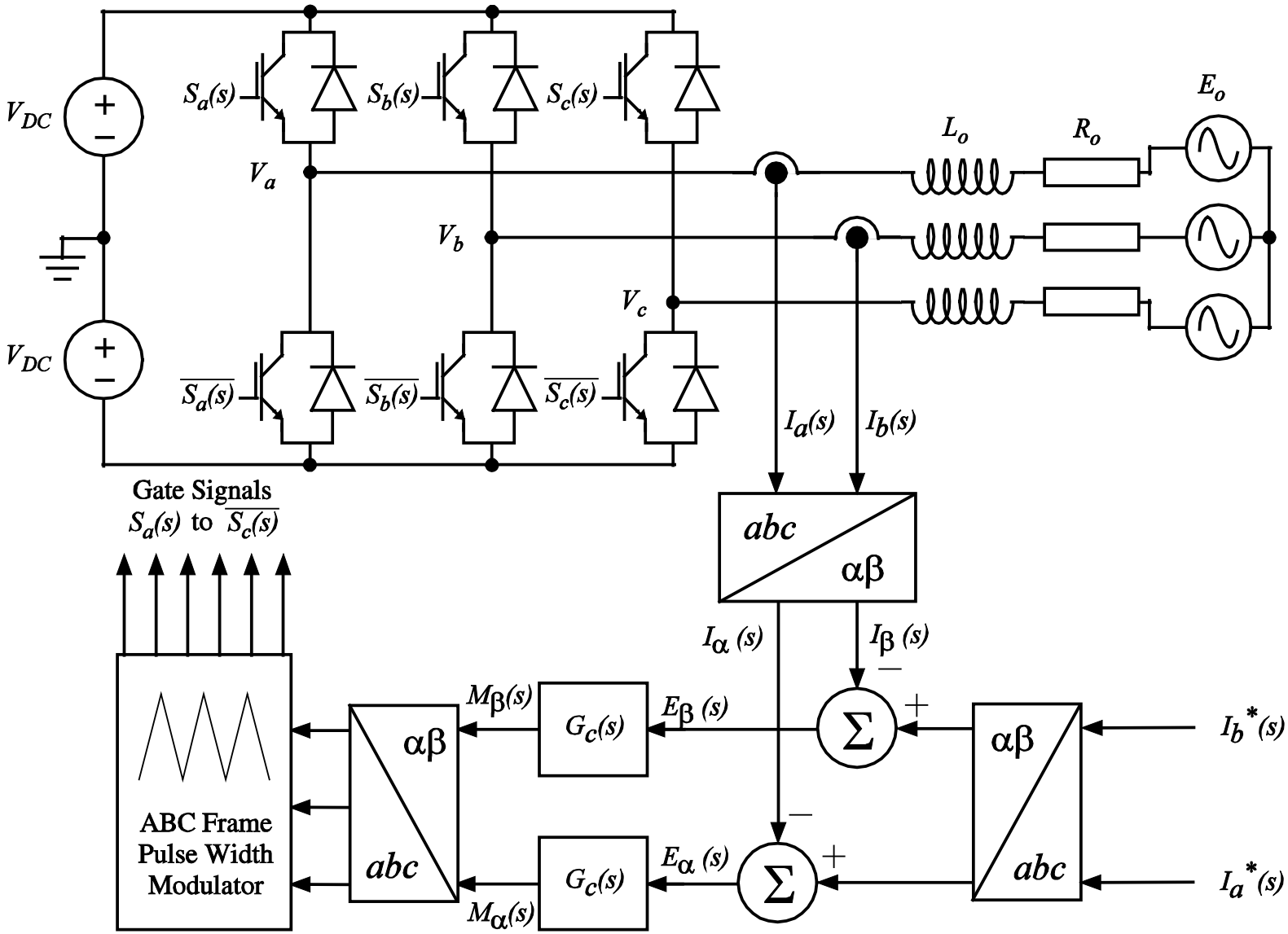
Orthogonal Reference Frames: $\alpha\beta$ stationary frame

- The $(\alpha\beta)$ quantities are sinusoidal
- Hence gain maximisation approach is identical to the (abc) frame current regulator
- The regulator output is compatible with a direct space vector modulator – no need to indirectly calculate the phase C PWM command for SV implementation (but still need to decode SV output to phase leg switching commands)
- References can be defined in either the (abc) or the $(\alpha\beta)$ frames as suits application

Orthogonal Reference Frames: $\alpha\beta$ stationary frame



Orthogonal Reference Frames: $\alpha\beta$ stationary frame



Orthogonal Reference Frames: *dq* rotating frame

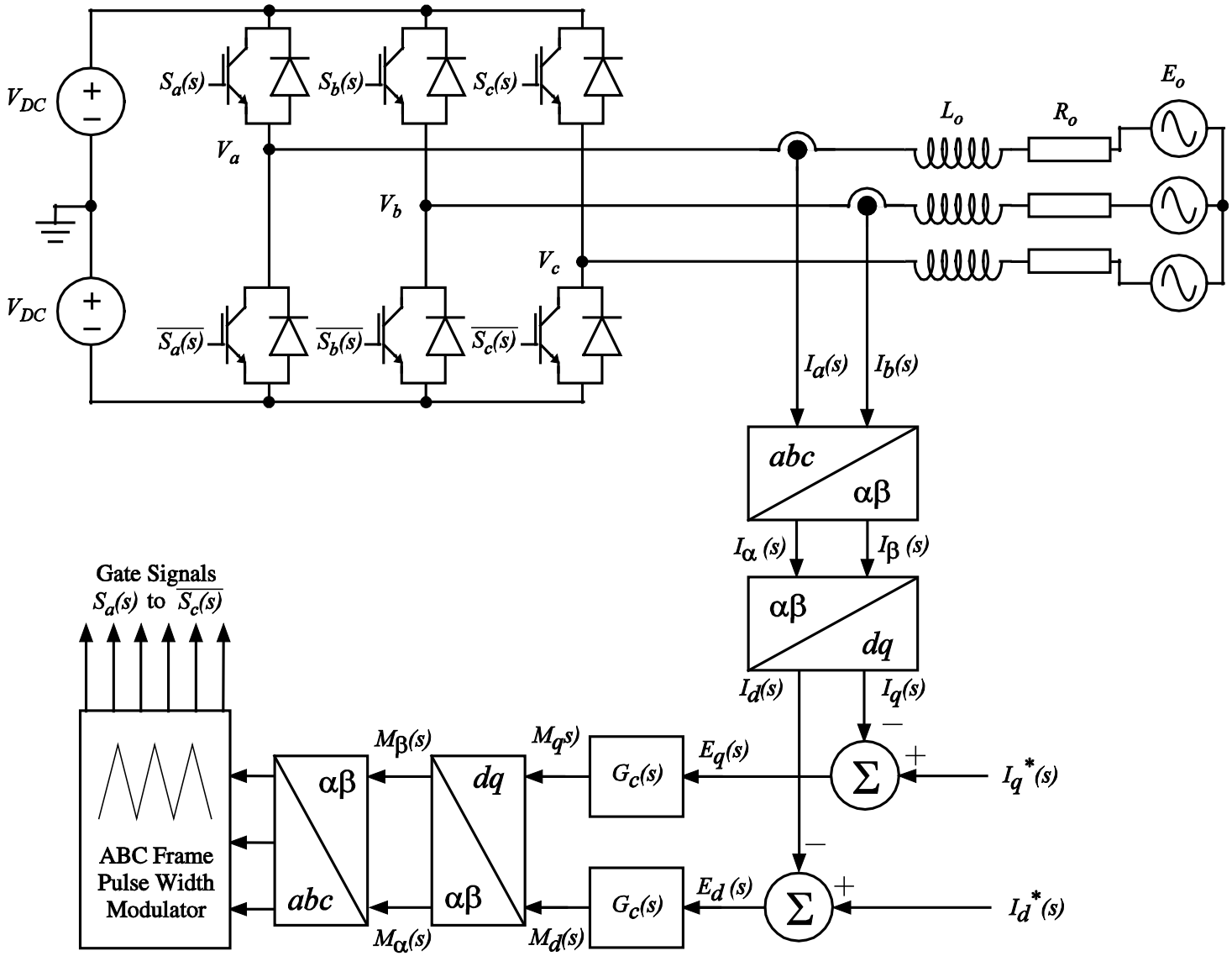
- Synchronous frame – AC quantities are transformed to equivalent DC quantities
- Park transform – $(\alpha\beta)$ to (dq)

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = T_{dq} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \qquad \theta = \omega t$$

- Inverse Park transform:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = T_{dq}^{T} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \qquad \theta = \omega t$$

Orthogonal Reference Frames: *dq* rotating frame



Reference Frames: regulator transformation

Stationary frame $\alpha\beta$ controller

$$\begin{bmatrix} K_P\left(1+\frac{1}{s\tau_i}\right) & 0 \\ 0 & K_P\left(1+\frac{1}{s\tau_i}\right) \end{bmatrix}$$

Rotating frame DQ controller

$$\begin{bmatrix} K_P\left(1+\frac{1}{\tau_i}\frac{s}{s^2+\omega_o{}^2}\right) & \frac{1}{\tau_i}\frac{\omega_o}{s^2+\omega_o{}^2} \\ -\frac{1}{\tau_i}\frac{\omega_o}{s^2+\omega_o{}^2} & K_P\left(1+\frac{1}{\tau_i}\frac{s}{s^2+\omega_o{}^2}\right) \end{bmatrix}$$

Stationary Frame PI regulator transforms to Synchronous Frame Resonant

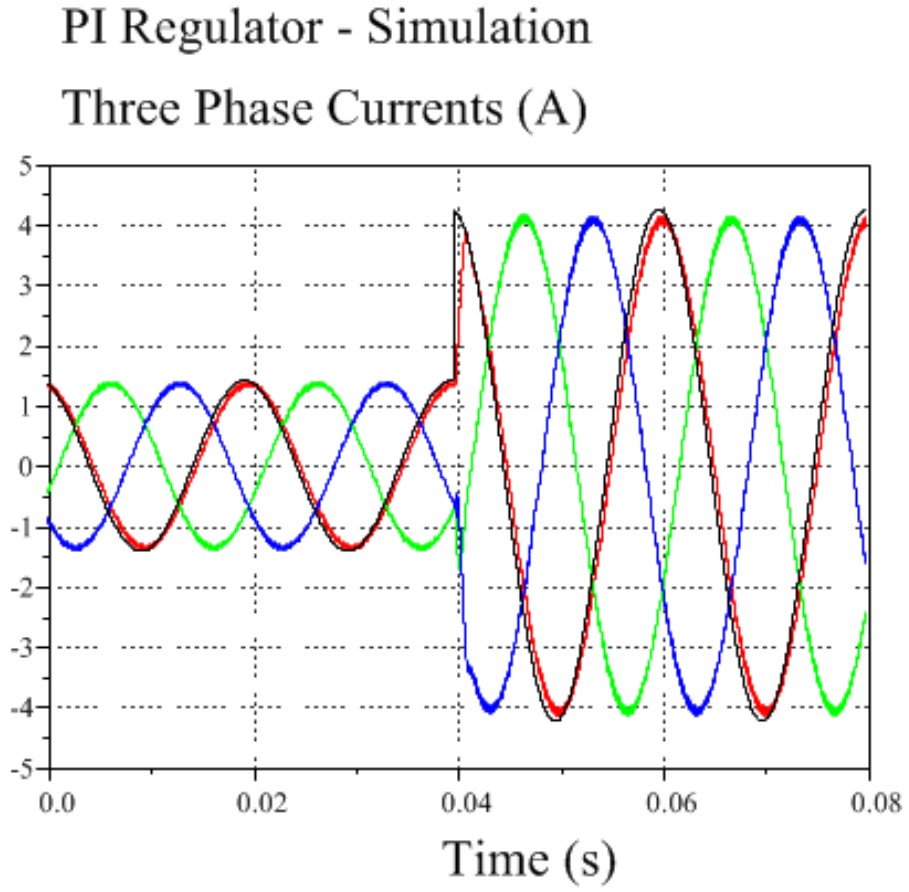
$$\begin{bmatrix} K_P\left(1+\frac{1}{\tau_i}\frac{s}{s^2+\omega_o{}^2}\right) & \frac{1}{\tau_i}\frac{\omega_o}{s^2+\omega_o{}^2} \\ -\frac{1}{\tau_i}\frac{\omega_o}{s^2+\omega_o{}^2} & K_P\left(1+\frac{1}{\tau_i}\frac{s}{s^2+\omega_o{}^2}\right) \end{bmatrix}$$

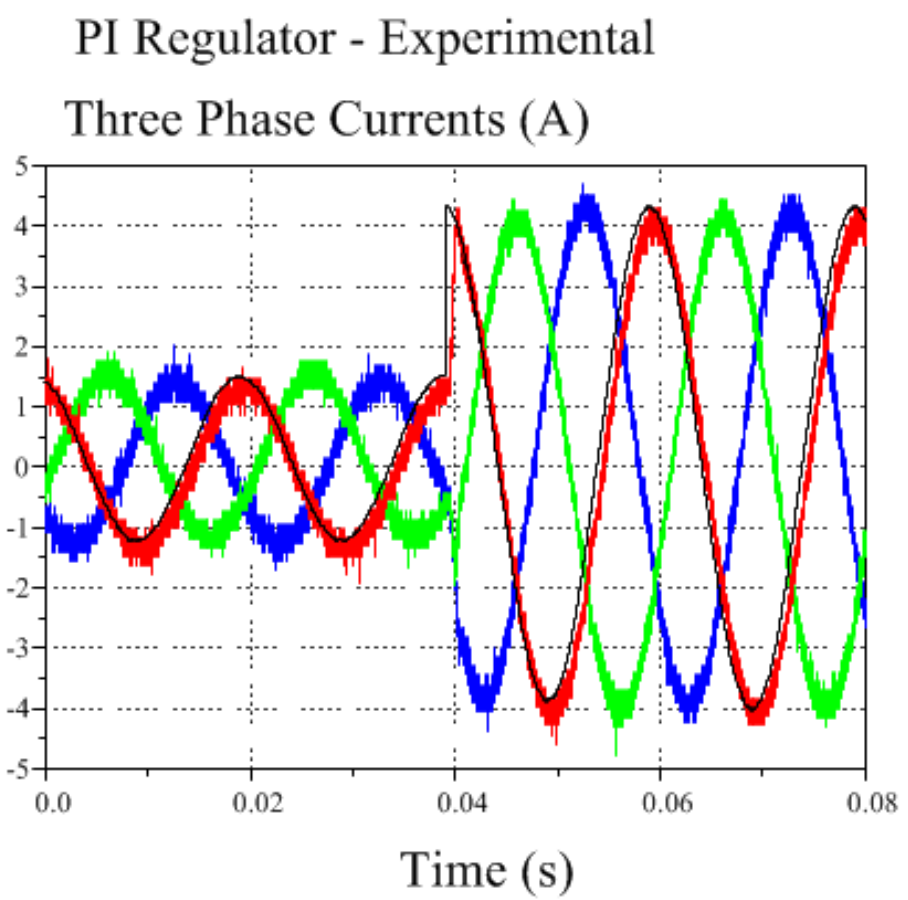
$$\begin{bmatrix} K_P\left(1+\frac{1}{s\tau_i}\right) & 0 \\ 0 & K_P\left(1+\frac{1}{s\tau_i}\right) \end{bmatrix}$$

Synchronous Frame PI regulator transforms to Stationary Frame Resonant

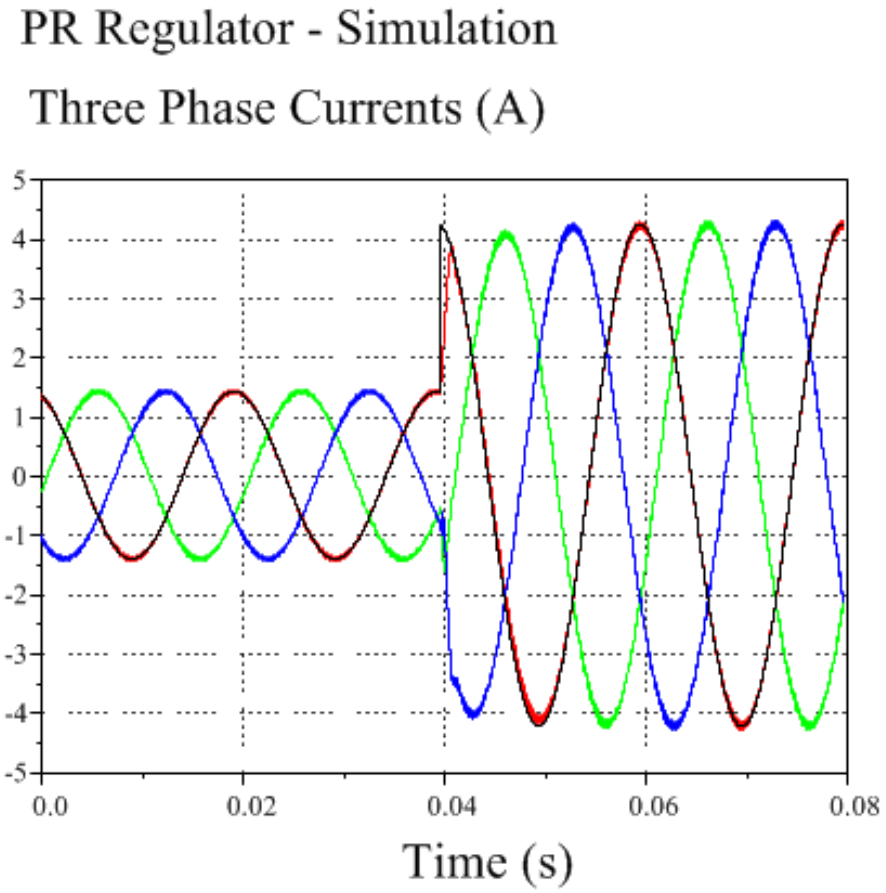
Gains are the same in both frames of reference and hence can be calculated as before

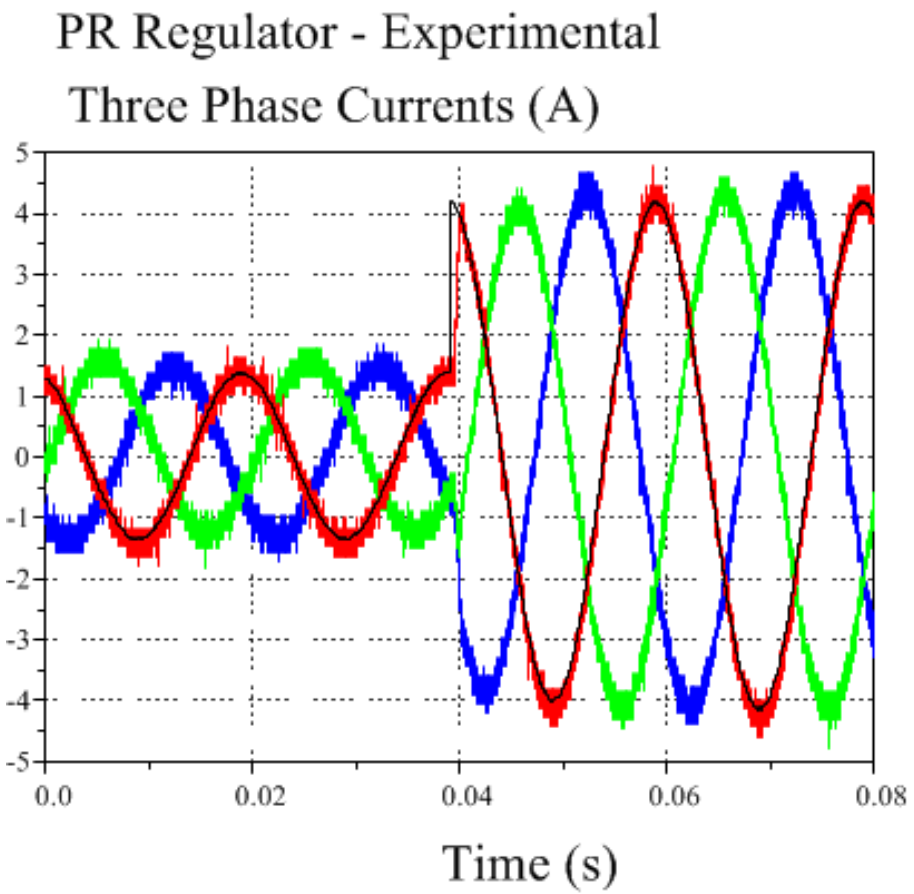
Simulated and Experimental PR Current Responses





Simulated and Experimental PR Current Responses





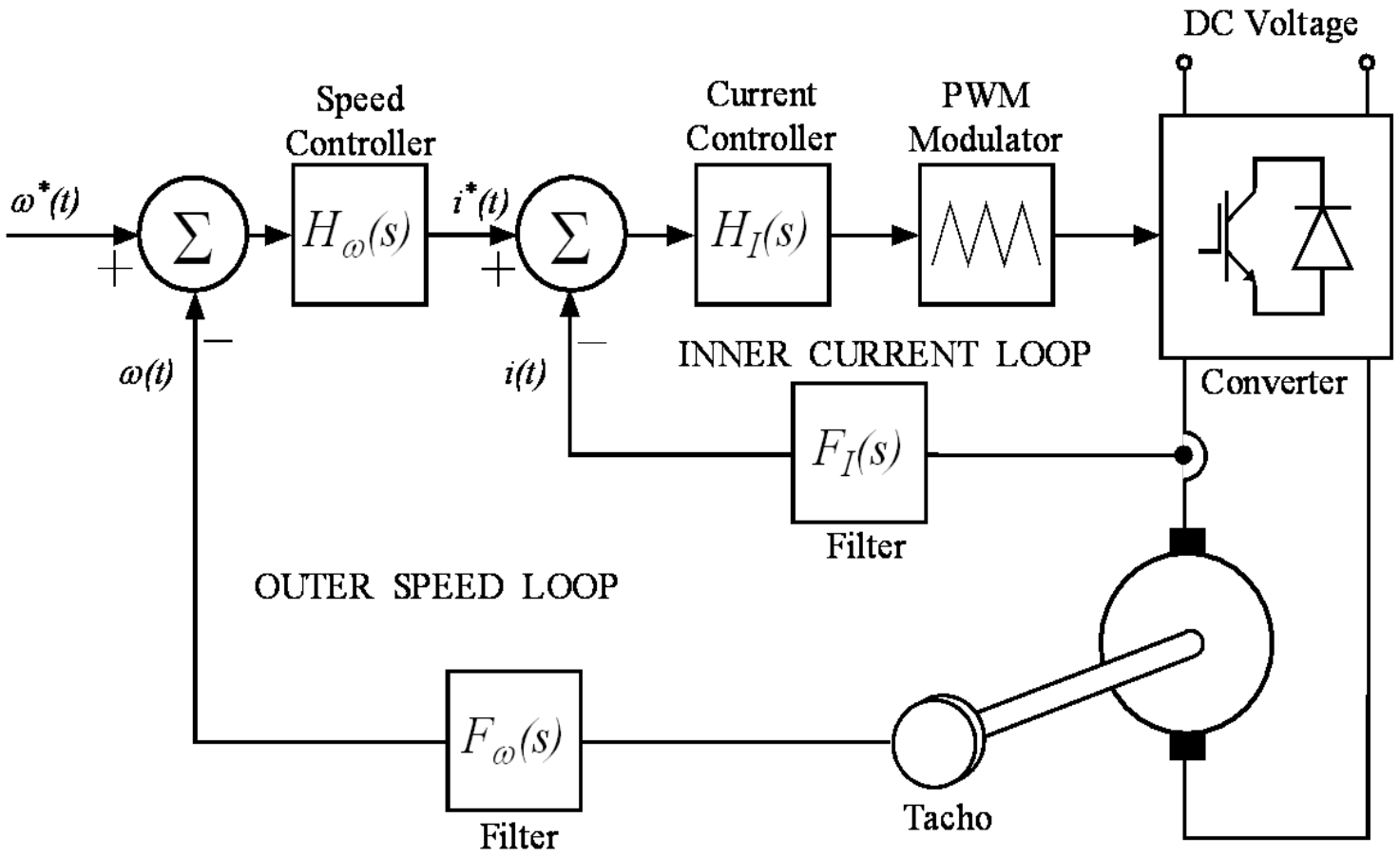
Example: AC Induction Motor Variable Speed Drive

$$i_{qs}^{*}(s) = k_{p\omega}\left(1+\frac{1}{s\tau_{r\omega}}\right)\left(\omega_r^{*}(s)-\omega_r(s)\right)$$

Torque Controller

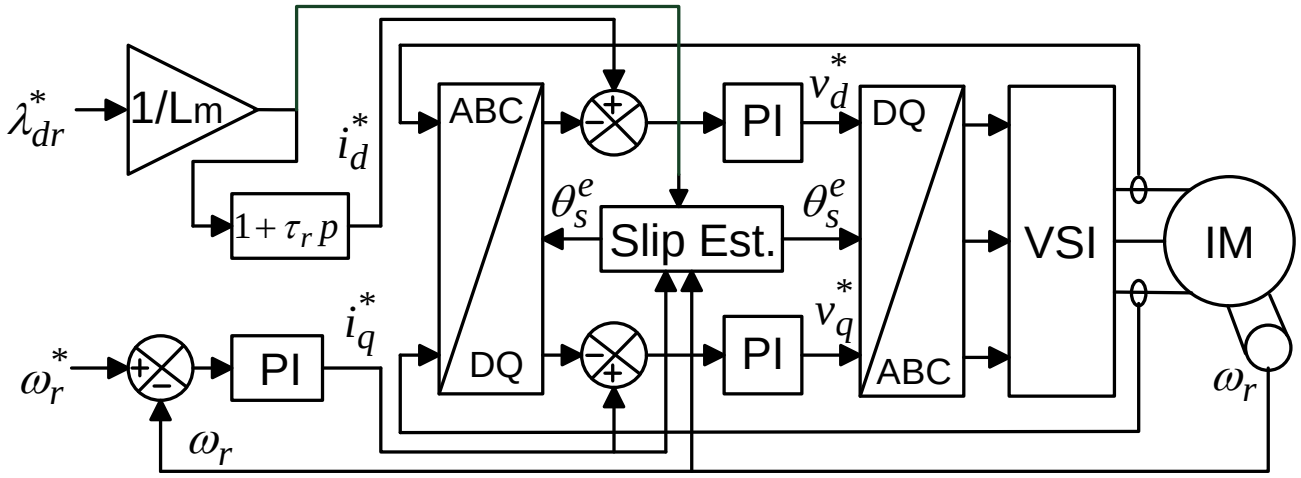
$$i_{ds}^{*}(s) = (1+\tau_r s)\frac{\lambda_{dr}^{*}(s)}{L_m}$$

Flux Controller



Example: AC Induction Motor Variable Speed Drive

Synchronous Frame PI Regulator



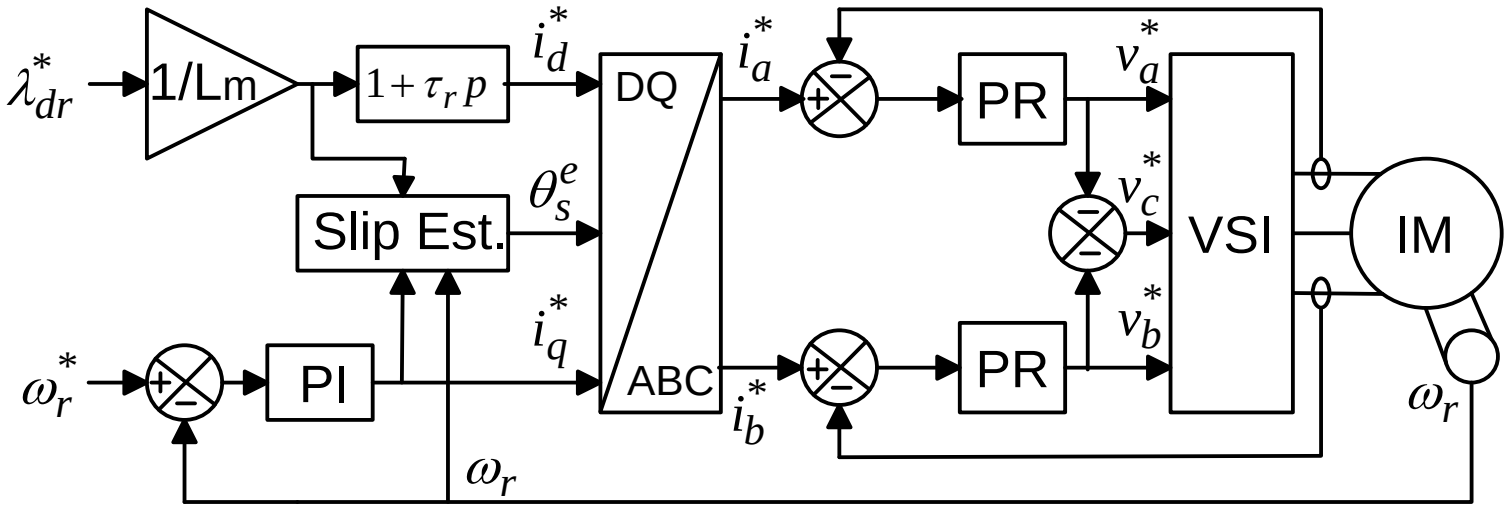
$$\begin{bmatrix} v_d^*(s) \\ v_q^*(s) \end{bmatrix} = K_p\left(1+\frac{1}{s\tau_i}\right)\begin{bmatrix} i_d^*(s)-i_d(s) \\ i_q^*(s)-i_q(s) \end{bmatrix}$$

$$\begin{bmatrix} v_a^* \\ v_b^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}\begin{bmatrix} \cos\theta_s^e & -\sin\theta_s^e \\ \sin\theta_s^e & \cos\theta_s^e \end{bmatrix}\begin{bmatrix} v_d^* \\ v_q^* \end{bmatrix}$$

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\theta_s^e & \sin\theta_s^e \\ -\sin\theta_s^e & \cos\theta_s^e \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/\sqrt{3} & 2/\sqrt{3} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

Example: AC Induction Motor Variable Speed Drive

Stationary Frame PR Regulator

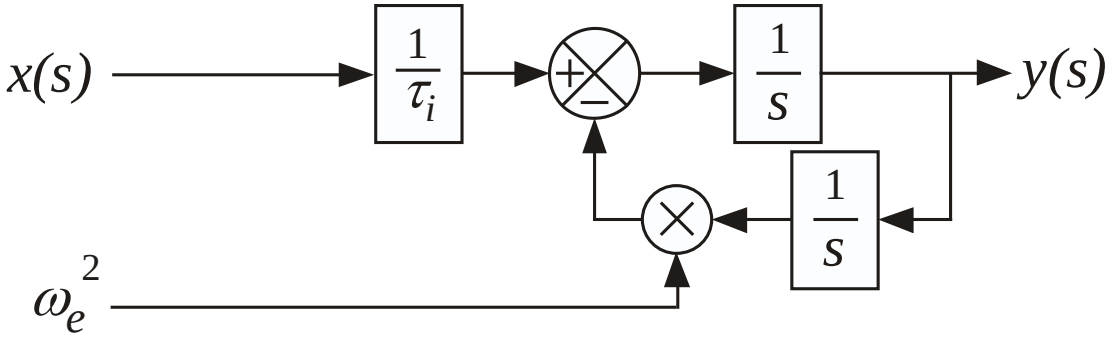


$$\begin{bmatrix} v_a^*(s) \\ v_b^*(s) \end{bmatrix} = K_p \left(1 + \frac{1}{\tau_i} \frac{s}{s^2 + \omega_e^2} \right) \begin{bmatrix} i_a^*(s) - i_a(s) \\ i_b^*(s) - i_b(s) \end{bmatrix}$$

Example: AC Induction Motor Variable Speed Drive

Stationary Frame PR Regulator

- Resonance was implemented using the Second Order Generalised Integrator:



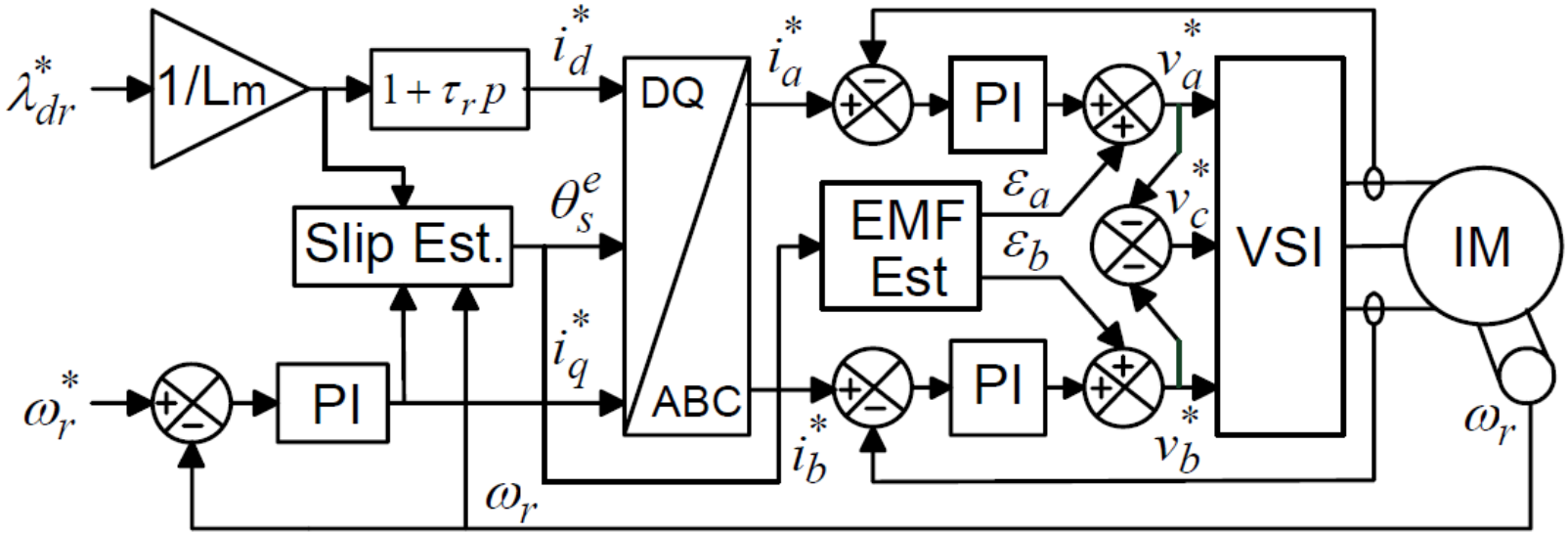
- Provides the transfer function:

$$\frac{y(s)}{x(s)} = \frac{1}{\tau_i}\frac{s}{s^2 + \omega_e^2}$$

- Advantage of this form is that the integrators are unaffected by changes in resonance frequency, simplifying the implementation in a DSP.

Example: AC Induction Motor Variable Speed Drive

Stationary Frame PI Regulator with EMF_{est}

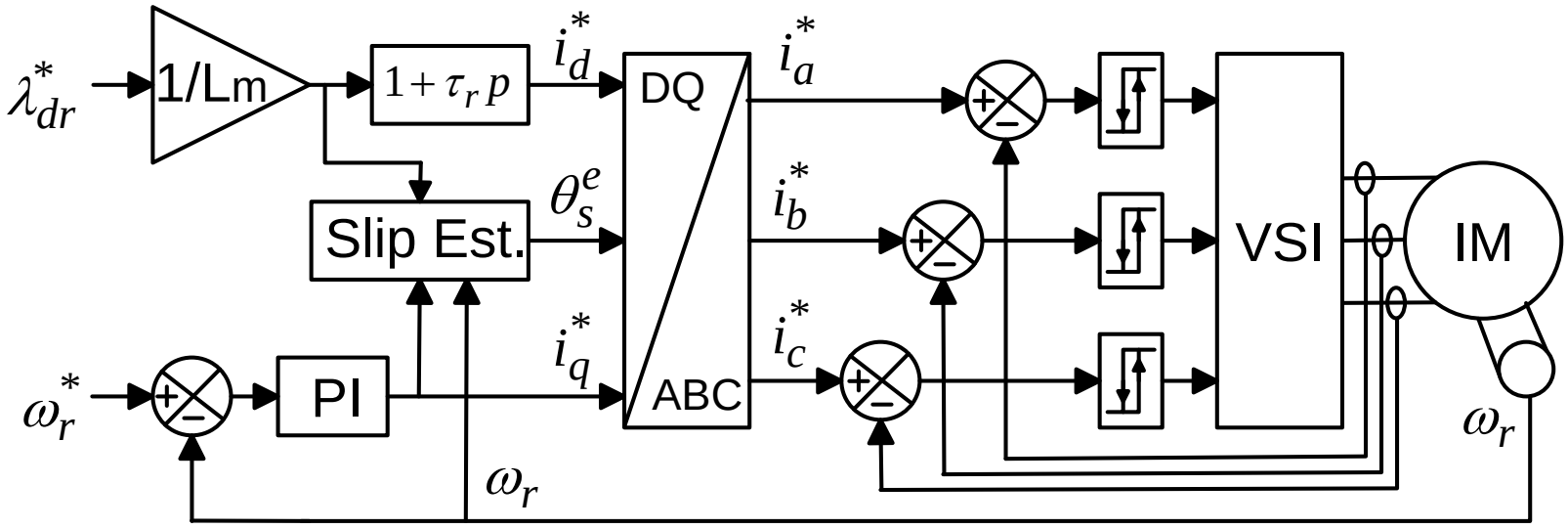


$$\begin{bmatrix} v_a^*(s) \\ v_b^*(s) \end{bmatrix} = K_p\left(1+\frac{1}{s\tau_i}\right)\begin{bmatrix} i_a^*(s)-i_a(s) \\ i_b^*(s)-i_b(s) \end{bmatrix} + \begin{bmatrix} \varepsilon_{a(est)} \\ \varepsilon_{b(est)} \end{bmatrix}$$

$$\varepsilon_{a(est)} \approx V_{nom}\frac{\omega_r}{\omega_{nom}}\angle\theta_s^e + 90^0$$

Example: AC Induction Motor Variable Speed Drive

Variable Band Hysteresis (digital implementation)



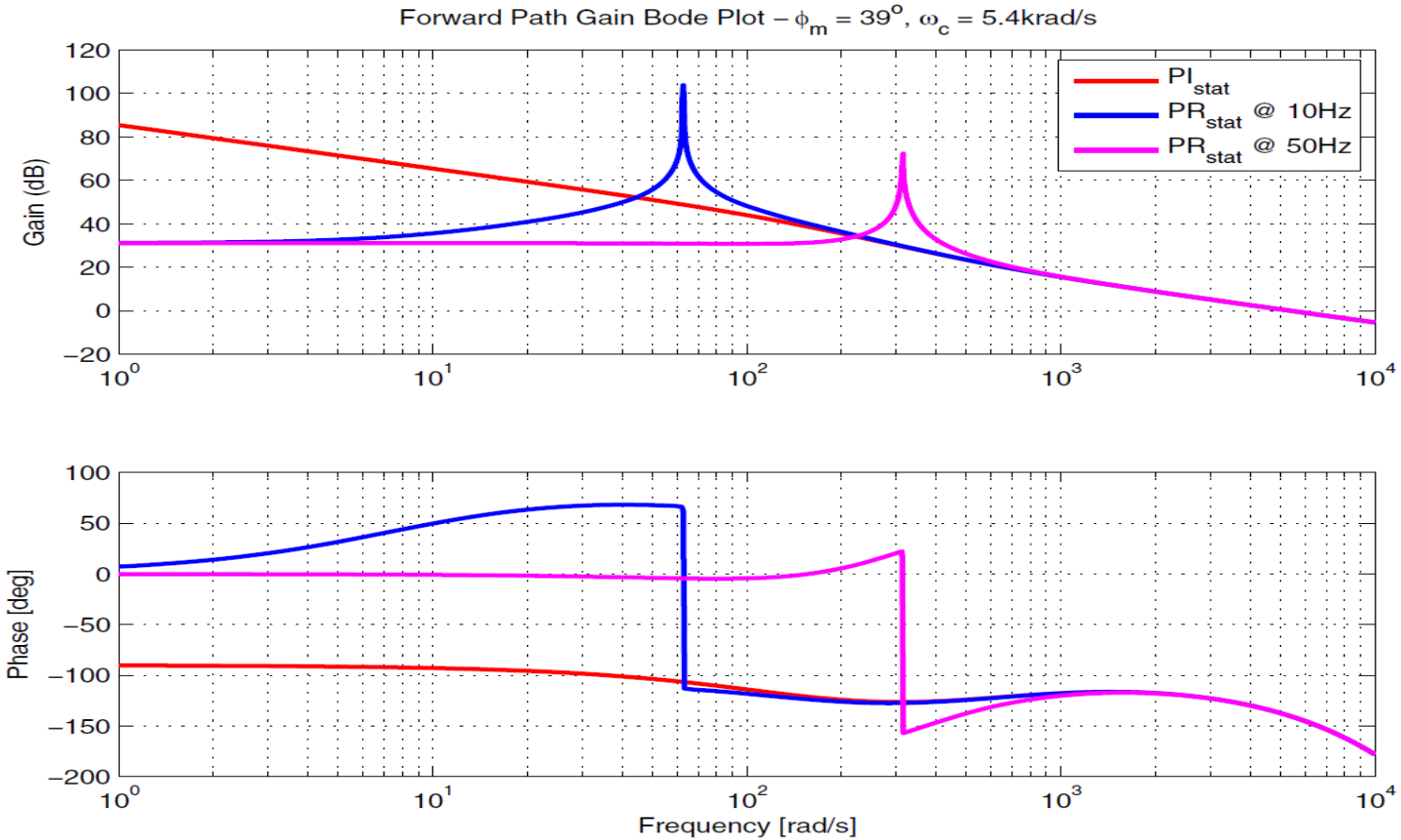
$$i_x^* - i_x < -i_h \qquad v_x \rightarrow +V_{DC}$$

$$i_x^* - i_x > i_h \qquad v_x \rightarrow -V_{DC}$$

Implemented digitally for this example at 100 kHz update rate, average switching frequency of approx 5 kHz

Example: AC Induction Motor Variable Speed Drive

Gains matched for the three different strategies



Gains can be set to same maximum gains irrespective of current regulation strategy – sampling and transport delay are the limiting factors

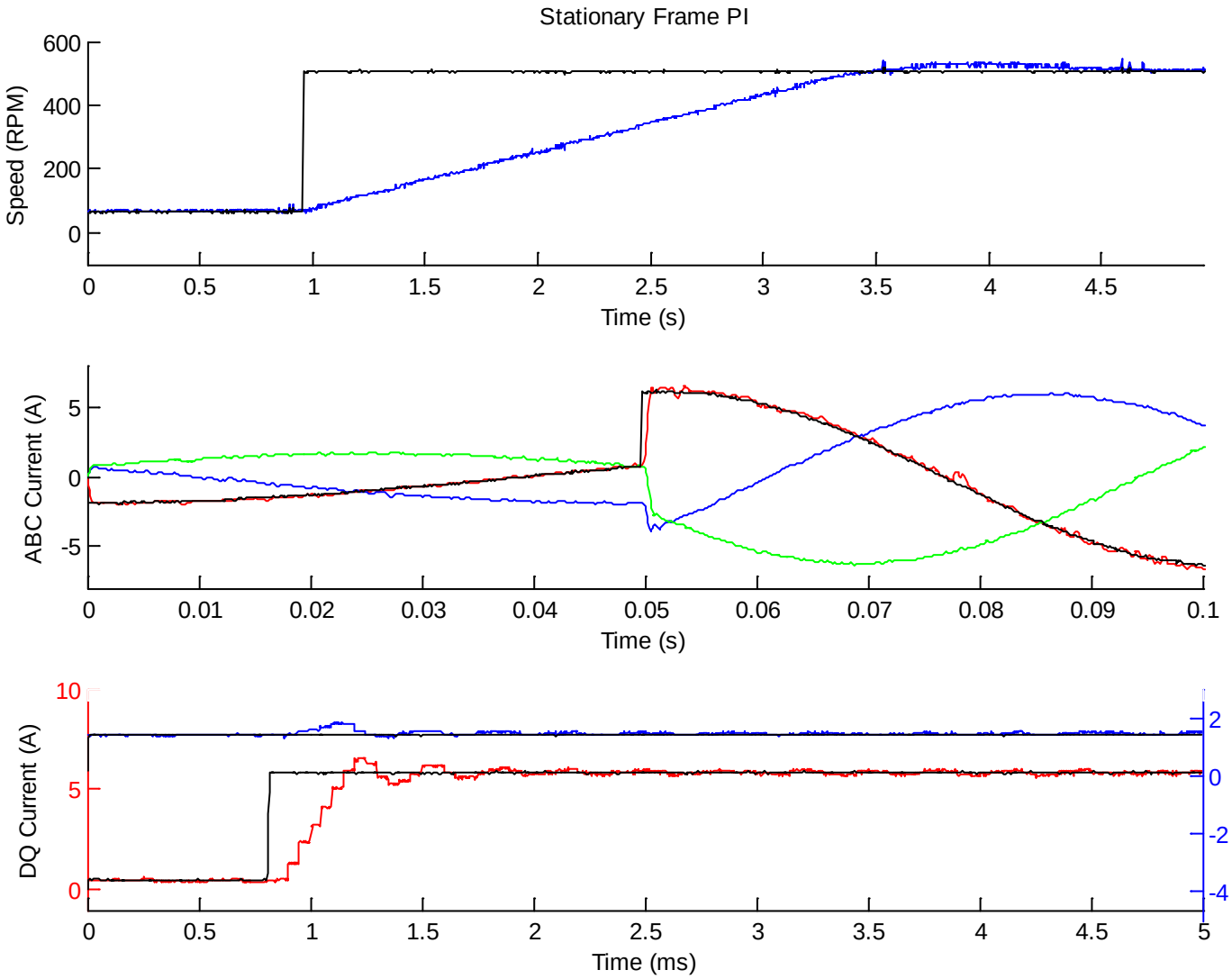
Example: AC Induction Motor Variable Speed Drive

Experimental System

- 15 kW 4 phase leg inverter (3 phase VSI plus DC bus chopper)
- DSP Controller: Texas Instruments TMS320F2810 fixed point DSP
- 3 phase 5kW Induction Motor
  - Quadrature pulse encoder to measure speed and direction
  - Coupled DC motor and drive to provide variable loading

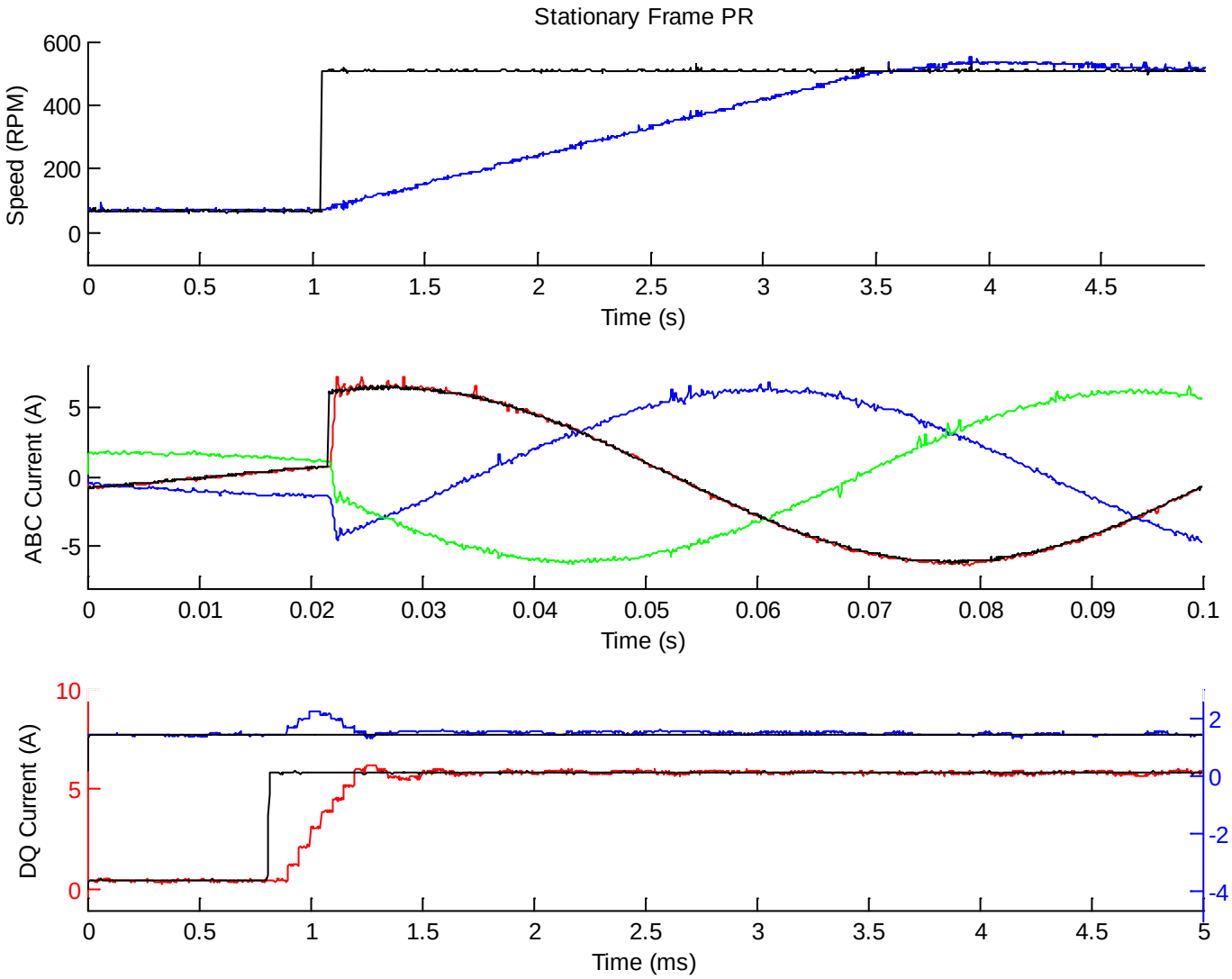
Parameter		Value
Induction Motor (4 pole)		3.2 kW, 415 $V_{l\text{-}l}$, 6.3 A, Y
IM	- r_s	3.24 Ω
	- r_r	2.6 Ω
	- $L_{ls} = L_s - L_m$	19 mH
	- $L_{lr} = L_r - L_m$	21.1 mH
	- L_m	504.4 mH
Inertia		0.65 kgm^2
System Load Dynamics		$1.2 + 0.0034\omega_r + 70e^{-6}\omega_r^2$
Controller	- $k_{p\omega}$	0.95
	- $\tau_{r\omega}$	0.034 sec
	- k_{pi}	0.7
	- τ_{ri}	0.00192 sec
	- Calc. Freq.	10 kHz
PWM Frequency		5 kHz
VSI DC Link Voltage		520 V

Example: AC Induction Motor Variable Speed Drive



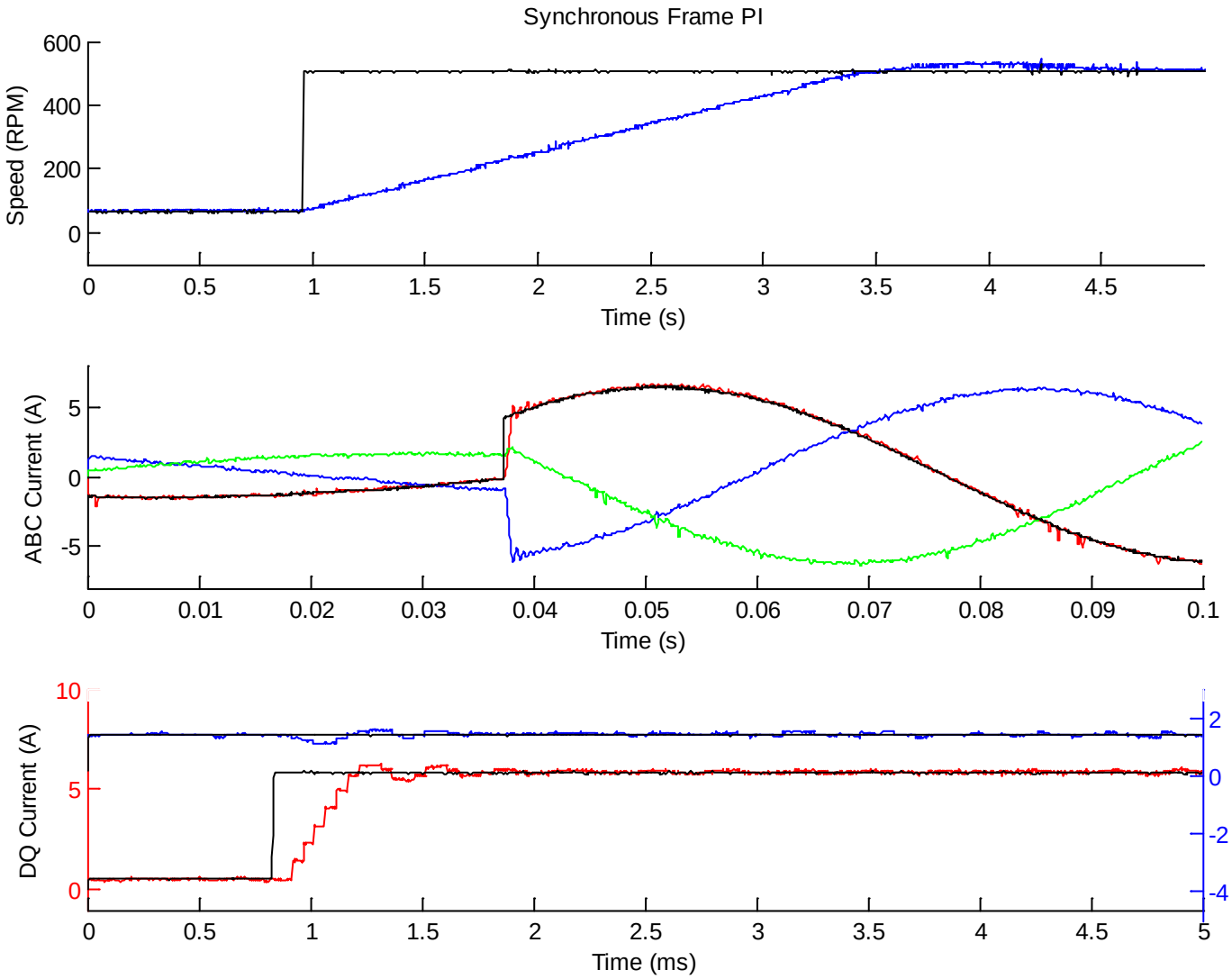
Experimental Current Step Change: $PI_{stationary}$

Example: AC Induction Motor Variable Speed Drive



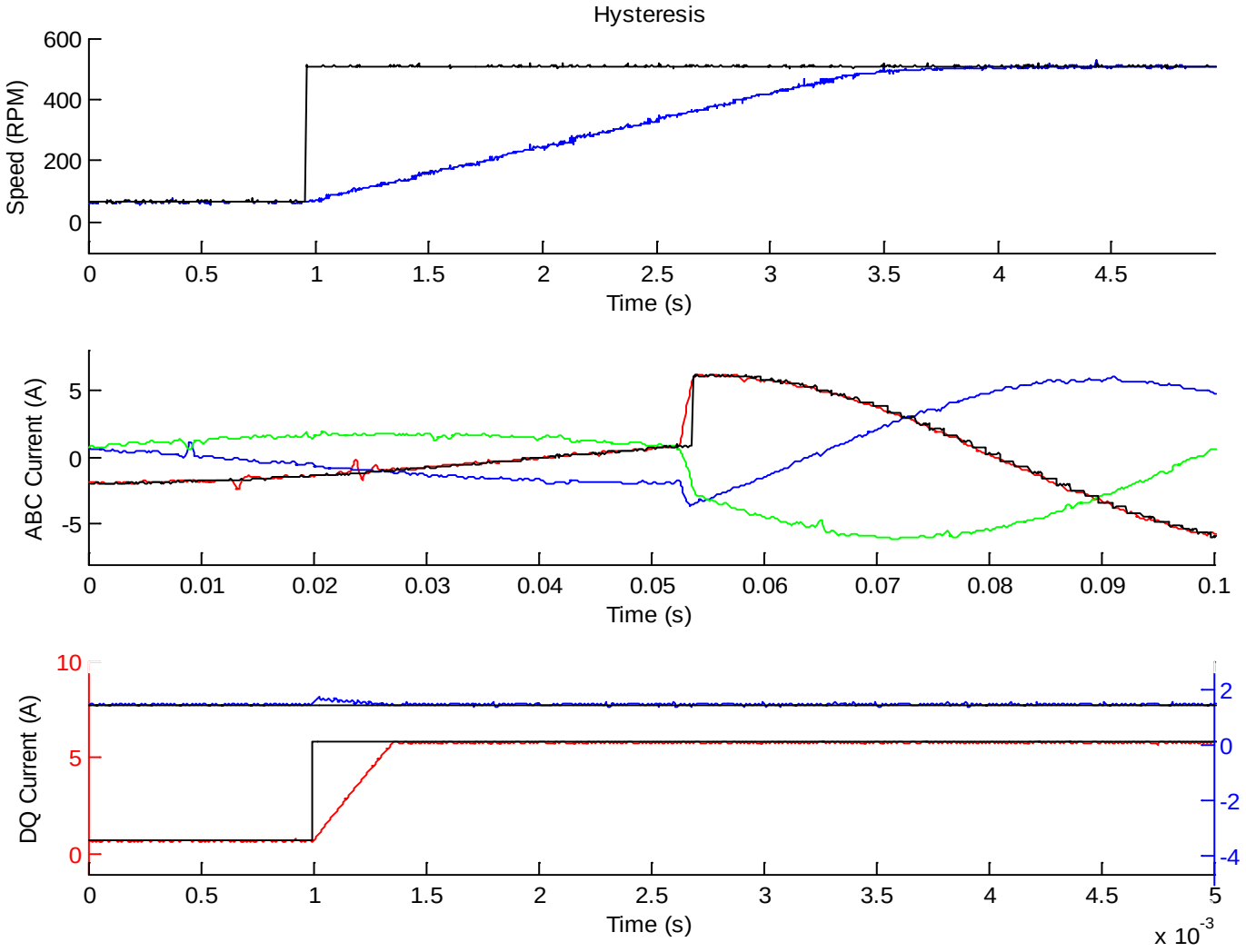
Experimental Current Step Change: $PR_{stationary}$

Example: AC Induction Motor Variable Speed Drive



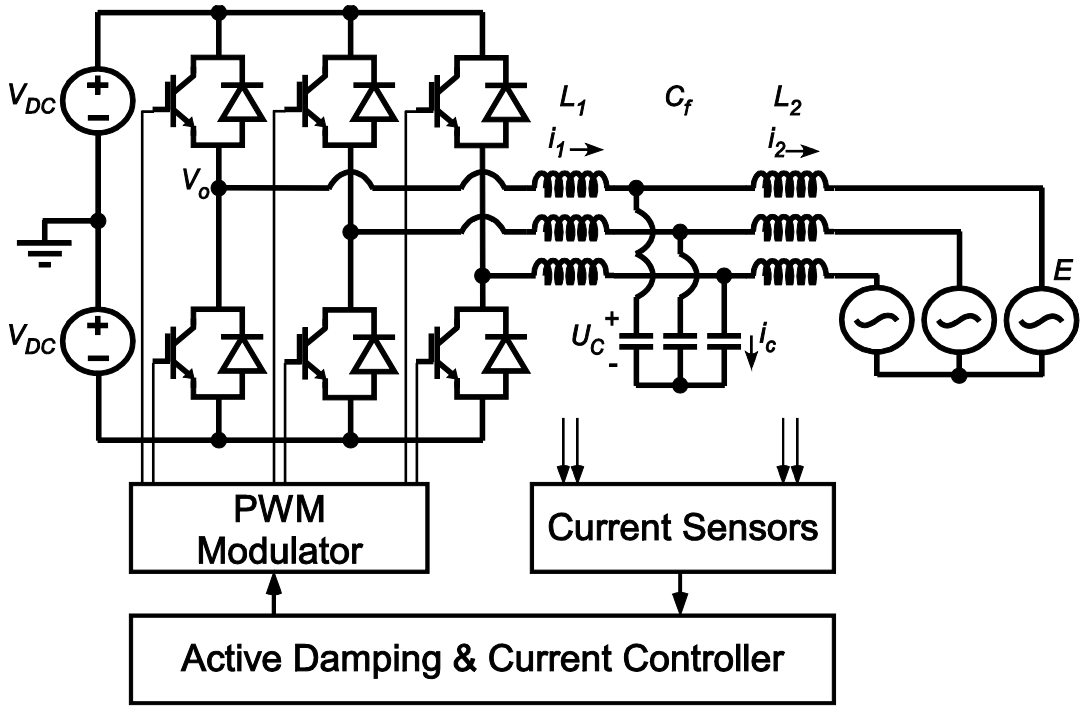
Experimental Current Step Change: $PI_{synchronous}$

Example: AC Induction Motor Variable Speed Drive



Experimental Current Step Change: Hysteresis

LCL Filters for AC Inverters



- LCL filter → Third order line filter for grid connected inverters
- Benefits of LCL filters (over L filters)
  - Greater attenuation, less size, less cost
- Major design challenges
  - LC network resonance, low harmonic impedance

Comparable L and LCL Filters

- L filter impedance

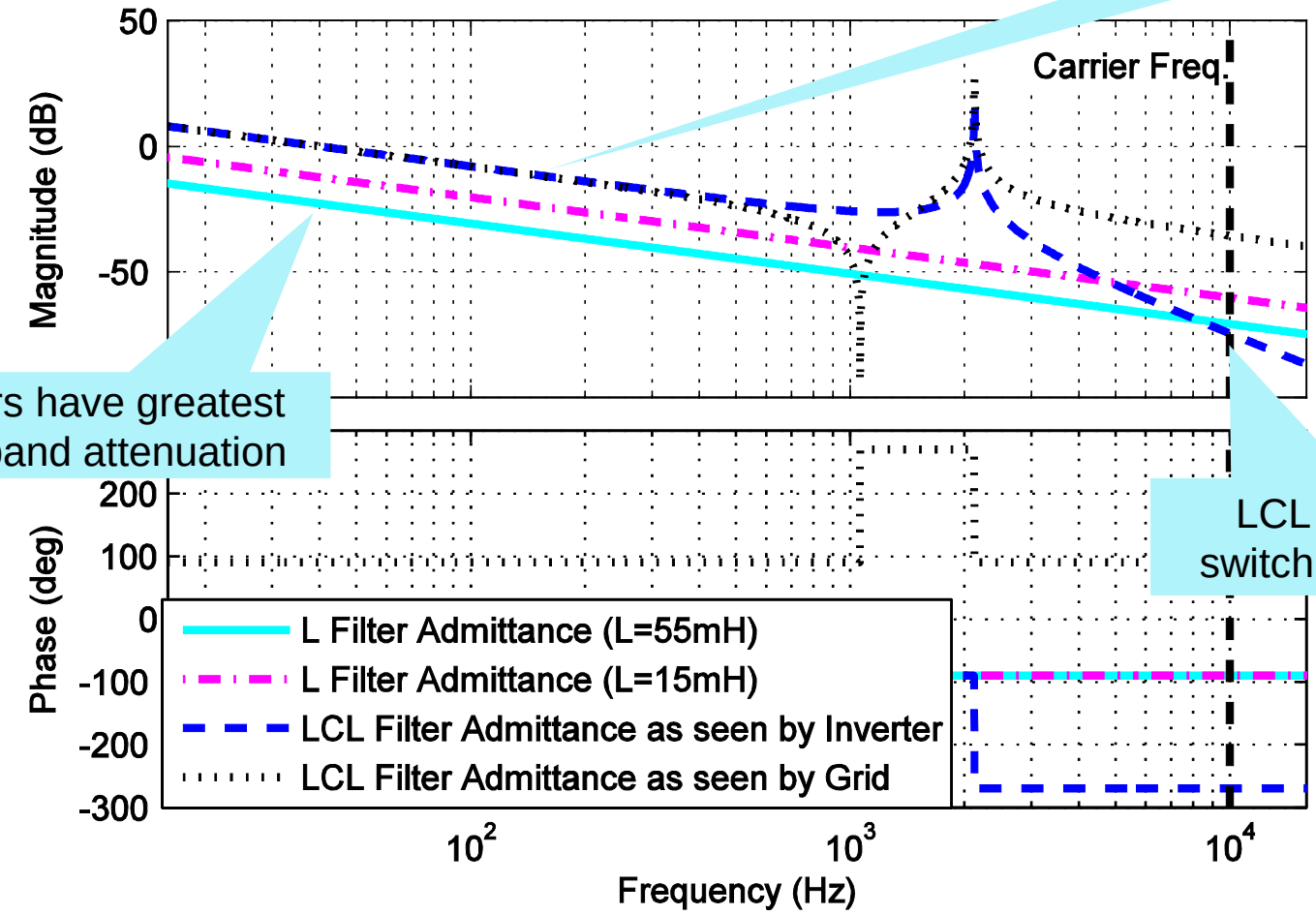
	@ **50Hz**	@ **10kHz**
L = 55mH	0.5pu	100pu
L = 16.5mH	0.15pu	30pu

- LCL filter impedance

L_1 = 3mH	@ **50Hz**	@ **10kHz**
C_f = 7.5uF	0.036pu	154pu
L_2 = 1mH		

- To meet IEEE519 → impedance >100pu @ 10kHz

L and LCL Filter Response



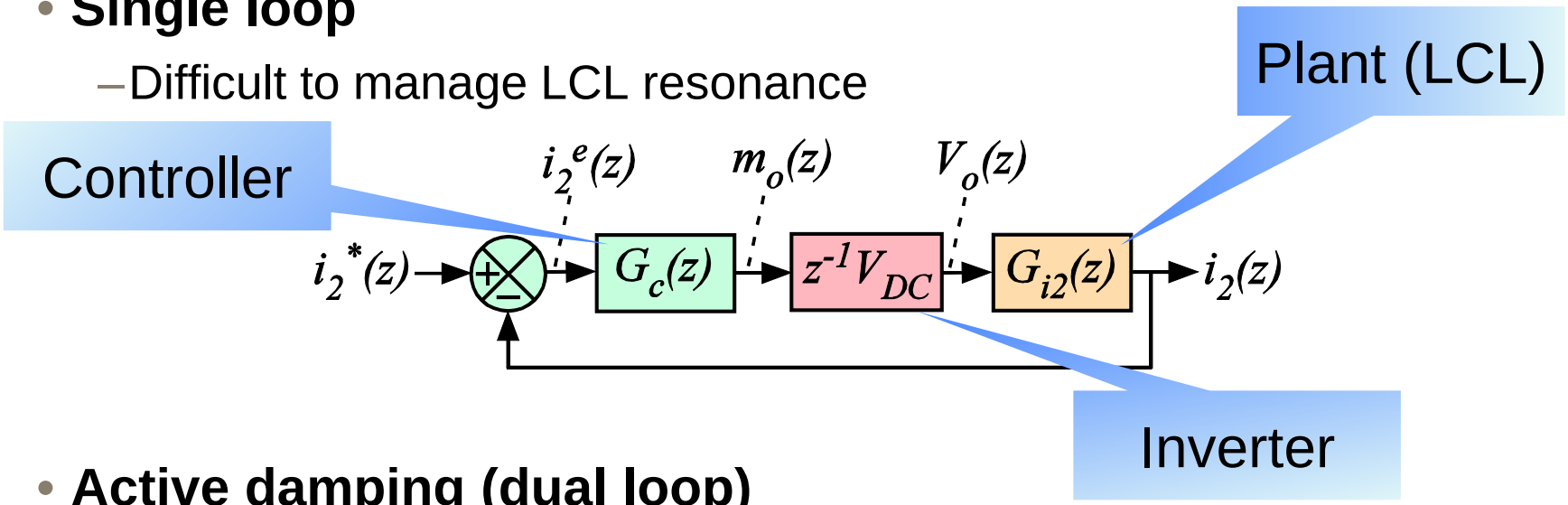
LCL filter with comparable switching frequency → can never achieve comparable baseband filtering

Control Loop Synthesis and Analysis

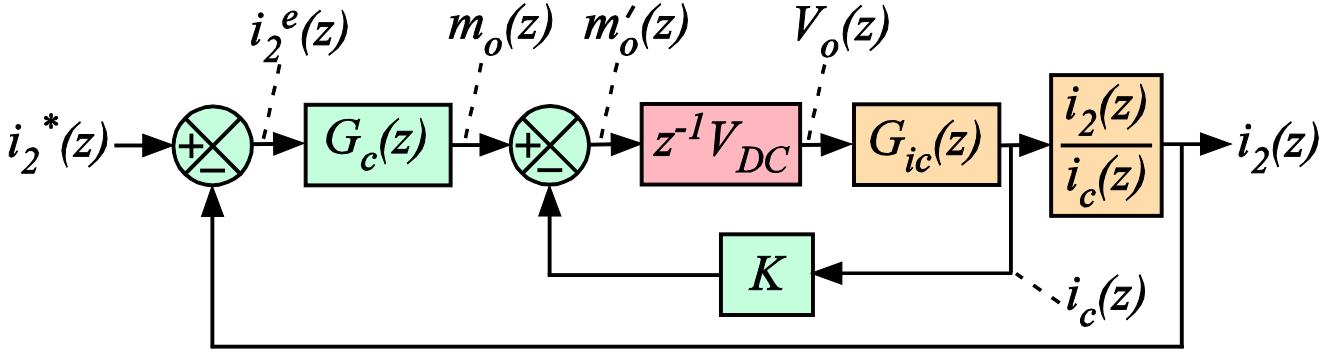
- LCL resonance damping:
  - Passively using series or shunt resistances
  - Actively using an additional state variable in the control law
- How to synthesise the current controller?
  - Analyse using discrete time models (i.e. state space or z-domain)
  - Sampling delay accounted for using a ZOH transformation of the LCL filter s-domain transfer function(s)
  - Transport delay accounted for using a z^{-1} shift operator
- Active damping is proposed in multiple forms:
  - Capacitor current feedback is the most commonly accepted
- Damping may not be required under some circumstances

LCL Filter Controller Options

- **Single loop**
  - Difficult to manage LCL resonance



- **Active damping (dual loop)**
  - Commonly feedback of i_c (or variant) for damping, through gain K



Which controller is suitable and when ? → use Bode analysis

L and LCL Filter Equations

- L Filter

$$i_2(s) = \frac{1}{sL}\left[V_o(s) - E(s)\right] = Y_L(s)\left[V_o(s) - E(s)\right]$$

$Y_L(s)$ is the filter admittance seen by the grid and converter for an L filter

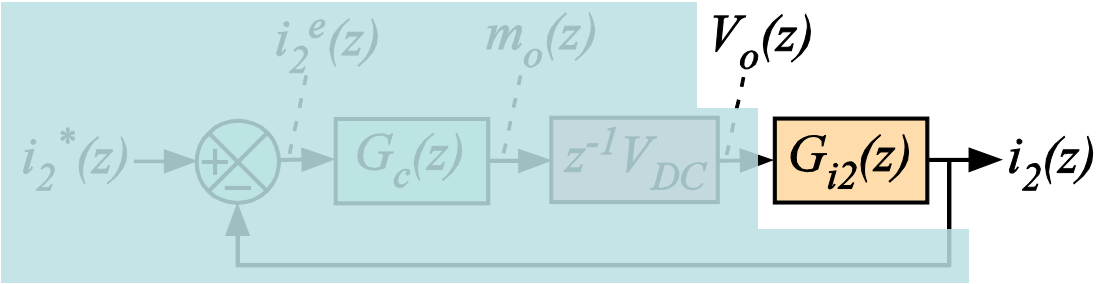
- LCL Filter

$$\begin{aligned} i_2(s) &= \frac{1}{L_1 L_2 C_f s}\frac{1}{\left(s^2 + \omega_{res}^2\right)}V_o(s) - \frac{1}{L_1 L_2 C_f}\frac{\left(L_1 C_f s^2 + 1\right)}{\left(s^2 + \omega_{res}^2\right)}E(s) \\ &= Y_v(s)V_o(s) + Y_G(s)E(s) \end{aligned}$$

$Y_v(s)$ and $Y_G(s)$ are the filter admittances seen by the converter and grid for an LCL filter, where $\omega_{res}^2 = \left(L_1 + L_2\right)\Big/\left(L_1 L_2 C_f\right)$

- NOTE : filter resistance neglected to model worst case undamped filter characteristic

Discrete Time Model – Single Loop Controller



- Plant transfer function (grid current)

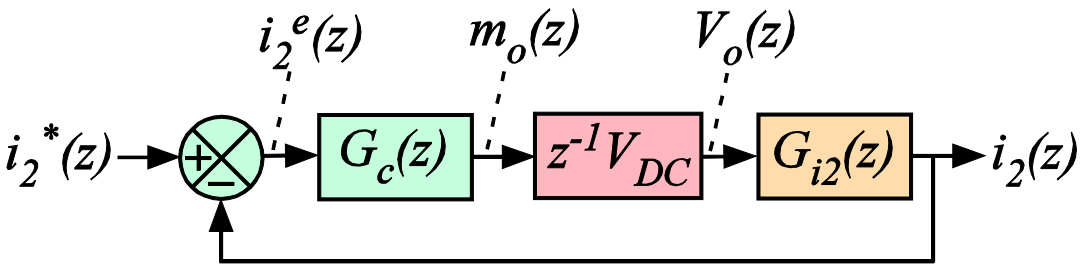
$$G_{i2}(s) = \frac{i_2(s)}{V_o(s)} = \frac{1}{sL_1}\frac{\gamma_{LC}^2}{\left(s^2 + \omega_{res}^2\right)}$$

- where $\omega_{res} = \sqrt{\frac{L_1 + L_2}{L_1 L_2 C_f}}$ and $\gamma_{LC} = \sqrt{\frac{1}{L_2 C_f}}$

- using ZOH discretization (sampled system)

$$G_{i2}(z) = \frac{i_2(z)}{V_o(z)} = \frac{T}{(L_1 + L_2)(z-1)} - \frac{\sin(\omega_{res}T)}{(L_1 + L_2)\omega_{res}} \times \frac{z-1}{z^2 - 2z\cos(\omega_{res}T) + 1}$$

Discrete Time Model – Single Loop Controller



- PR fundamental controller

$$G_c(s) = K_p\left(1 + \frac{1}{T_r}\frac{s}{s^2+\omega_0^2}\right) \Rightarrow G_c(z) = K_p\left(1 + \frac{1}{T_r}\frac{\sin(\omega_0 T)}{2\omega_0}\frac{z^2-1}{\left(z^2 - 2z\cos(\omega_0 T)+1\right)}\right)$$

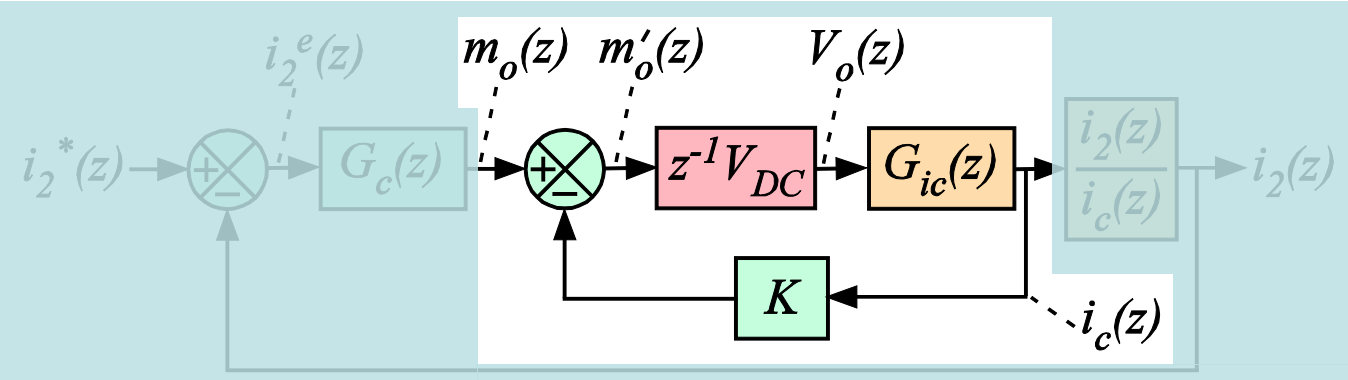
- Inverter model with PWM transport delay

$$V_o(z) = z^{-1}V_{DC}m_o(z)$$

- Open loop forward path equation (without active damping)

$$\frac{i_2(z)}{i_2^e(z)} = z^{-1}V_{DC}G_c(z)G_{i2}(z)$$

Discrete Time Model – Dual Loop Controller



- Inner loop plant transfer function (capacitor current)
  - Active damping

$$G_{ic}(s) = \frac{i_c(s)}{V_o(s)} = \frac{1}{sL_1}\frac{s^2}{\left(s^2 + \omega_{res}^2\right)}$$

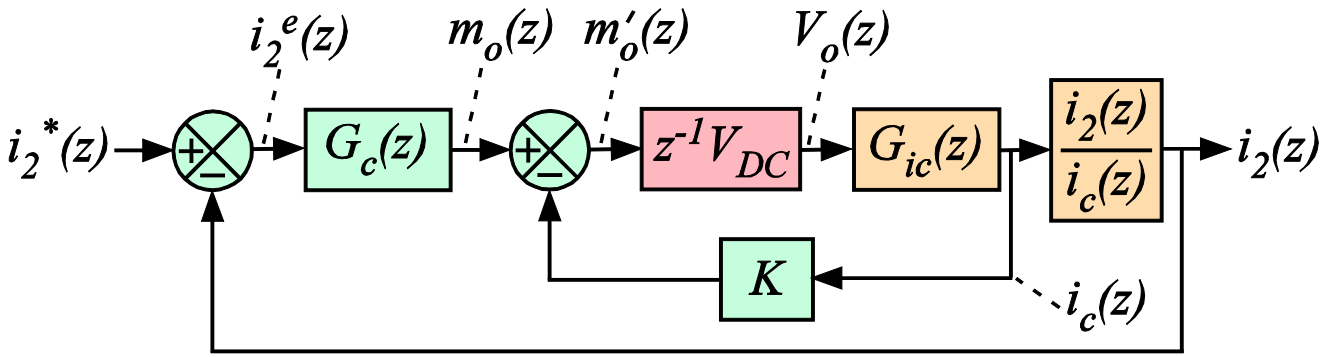
- Using ZOH discretization (sampled system)

$$G_{ic}(z) = \frac{i_c(z)}{V_o(z)} = \frac{\sin(\omega_{res}T)}{\omega_{res}L_1} \times \frac{z-1}{z^2 - 2z\cos(\omega_{res}T) + 1}$$

- Closed inner loop with gain K (including inverter model)

$$\frac{i_c(z)}{m_o(z)} = \frac{V_{DC}G_{ic}(z)}{z + KV_{DC}G_{ic}(z)}$$

Discrete Time Model – Dual Loop Controller



- Equation relaying i_2 to i_c
  - Impulse invariant discretization (delay already in model)

$$\frac{i_2(s)}{i_c(s)} = \frac{G_{i2}(s)}{G_{ic}(s)} = \frac{\gamma_{LC}^2}{s^2}$$

Zimp

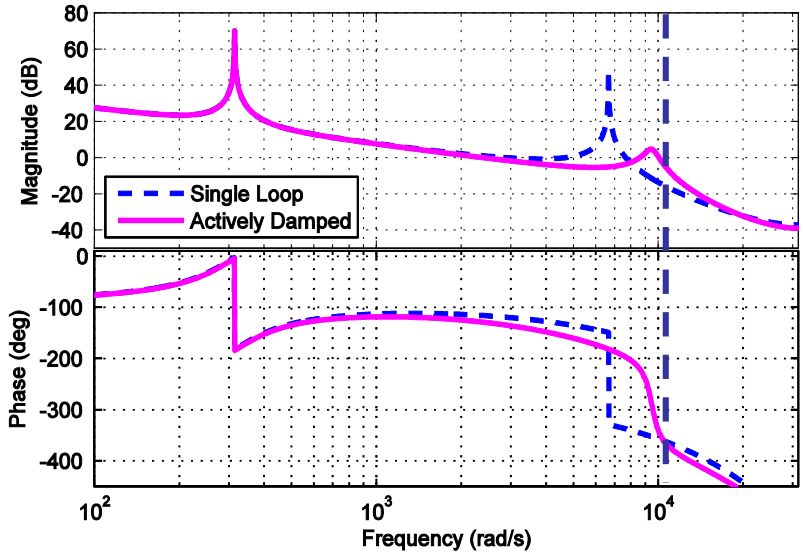
$$\frac{i_2(z)}{i_c(z)} = \frac{\gamma_{LC}^2 T^2 z}{(z-1)^2}$$

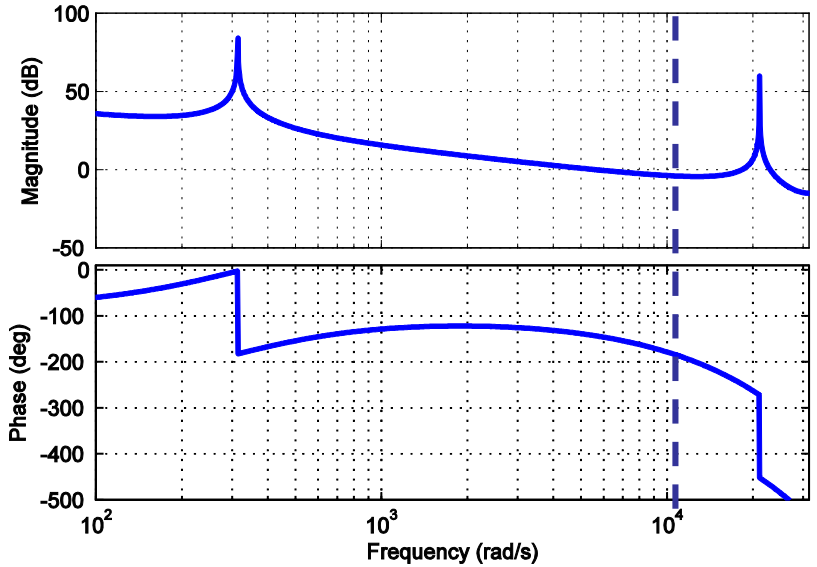
- Overall open loop forward path equation with active damping

$$\frac{i_2(z)}{i_2^e(z)} = G_c(z) \times \frac{i_c(z)}{m_o(z)} \times \frac{i_2(z)}{i_c(z)}$$

$$= G_c(z) \times \frac{V_{DC} G_{ic}(z) \left[i_2(z) / i_c(z) \right]}{z + K V_{DC} G_{ic}(z)}$$

LCL Filter Active Damping Regions





- Low LCL resonant frequency
  - -180^o phase jump with unbounded gain
  - Unstable without damping
  - Stabilised with active damping
- High LCL resonant frequency
  - -180^o phase jump occurs after phase crosses -180^o due to delay roll-off
  - Stable without damping

- Critical LCL frequency (ω_{crit}) → forward path phase reaches -180^o

Critical LCL Resonant Frequency

- To find ω_{crit}

$$\angle \frac{i_2}{i_2^e}\left(z = e^{j\omega T}\right) = \angle e^{-j\omega T} V_{DC} G_c\left(e^{j\omega T}\right) G_{i2}\left(e^{j\omega T}\right) = -\pi$$

- PR controller response well below crossover frequency
  - Phase contribution of controller is negligible: $\angle G_c\left(e^{j\omega T}\right) \approx 0$
- LCL resonance → no phase contribution until the resonant frequency is reached
  - Hence plant phase reduces to $\angle G_{i2}\left(e^{j\omega T}\right) = \angle \frac{1}{e^{j\omega T} - 1}$
- with substitution and manipulation

$$\angle \frac{i_2}{i_2^e}\left(z = e^{j\omega T}\right) = -\omega T - \frac{\pi}{2} - \frac{\omega T}{2} = -\pi \quad \Rightarrow \quad \omega_{crit} = \frac{\pi}{3T}$$

- Below ω_{crit} damping is required
- Above ω_{crit} single control loop is sufficient

Exploring the Active Damping Regions

- Closed loop poles → Required for root loci
- Using conventional $1+G(z)=0$, from forward path $G(z)$
  - Single loop controller

$$z + V_{DC} K_p G_{i2}(z) = 0$$

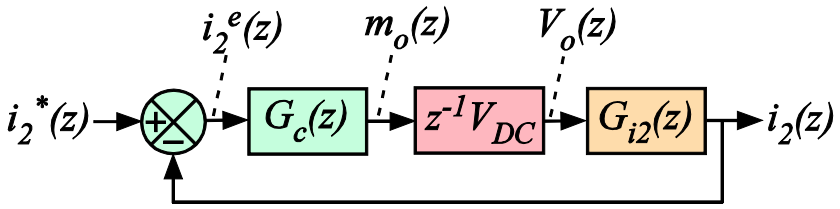
  - Inner loop of dual loop controller

$$z + V_{DC} K G_{ic}(z) = 0$$

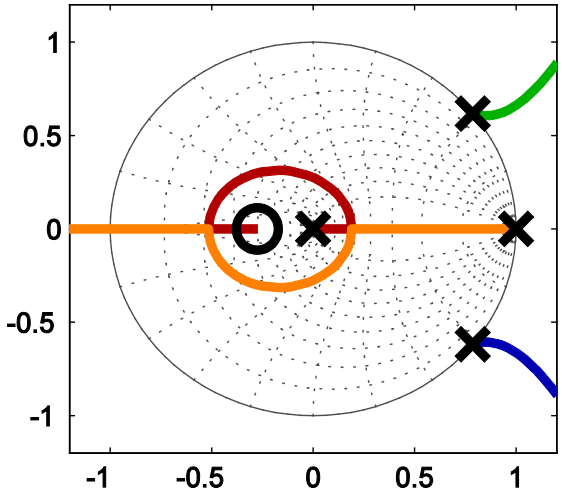
  - Overall dual loop controller

$$z + V_{DC} K G_{ic}(z) + V_{DC} K_P G_{ic}(z)\left[i_2(z) / i_c(z) \right] = 0$$

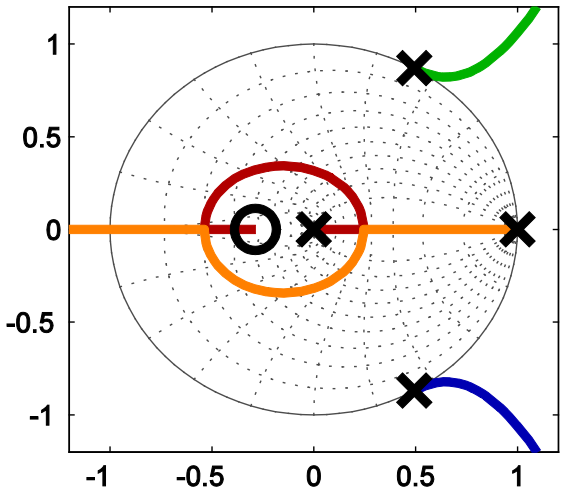
Single Loop (i_2) Controller Root Loci



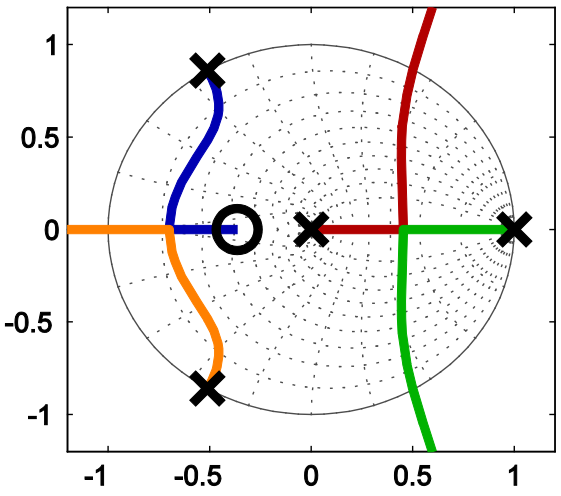
Low frequency



Critical frequency



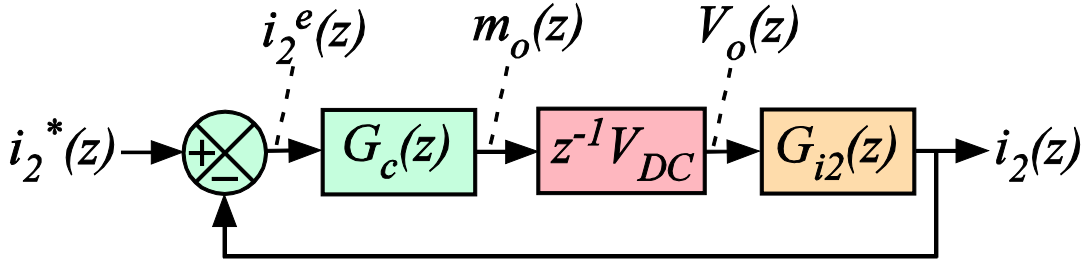
High frequency



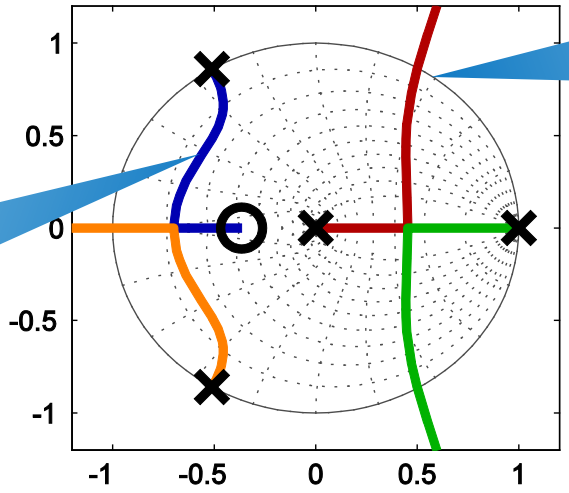
- No damping capability at critical or low frequencies
- Provides damping of poles at high frequency
  - No active damping required → agree with Bode
- All systems have loci which track unstable through $z = 0.5 \pm j\sqrt{3}/2$

Single Loop Controller when Resonance Above ω_{crit}

- Single loop controller is suitable
  - Stable, active damping unnecessary



- Gain limitation → Low frequency poles, not resonance
  - Same mechanism as L filters



Resonant poles will be damped

Low frequency poles cause instability

Gain Selection when Resonance Above ω_{crit}

- Using the same method as for standard L filters [Holmes et. al. 2009]
  - Resonance and controller → minimal magnitude and phase contribution at crossover frequency
  - Only low frequency model required → series inductance plant

$$\frac{i_2(z)}{i_2^e(z)} = z^{-1} V_{DC} K_p \frac{T}{(z-1)(L_1+L_2)}$$

  - Calculate crossover ω_c frequency for a desired phase margin φ_m

$$\angle \frac{i_2}{i_2^e}\left(z = e^{j\omega_c T}\right) = \angle \frac{V_{DC} K_p T}{(L_1+L_2)} \frac{1}{e^{j\omega_c T}\left(e^{j\omega_c T} - 1\right)}$$

$$= -\frac{\pi}{2} - \frac{\omega_c T}{2} - \omega_c T = -\frac{\pi}{2} - \frac{3}{2}\omega_c T$$

$$\varphi_m = \pi + \angle \frac{i_2}{i_2^e}\left(z = e^{j\omega_c T}\right) = \frac{\pi}{2} - \frac{3}{2}\omega_c T \quad \Rightarrow \quad \omega_c = \frac{\pi/2 - \varphi_m}{3T/2} \quad \Rightarrow \quad \omega_c = \frac{\pi/2 - \varphi_m}{T_d}$$

Gain Selection when Resonance Above ω_{crit}

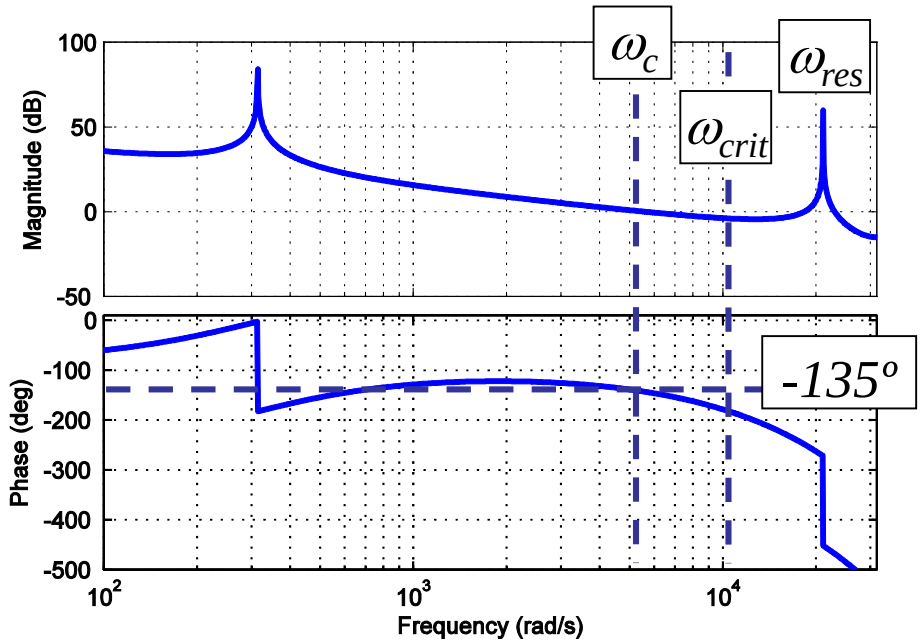
- Proportional gain is set to achieve unity gain at desired crossover frequency ω_c

$$K_p = \left| \frac{(L_1 + L_2)\left(e^{j\omega_c T} - 1\right)}{V_{DC} T} \right| \quad \Rightarrow \quad K_p \approx \frac{\omega_c (L_1 + L_2)}{V_{DC}}$$

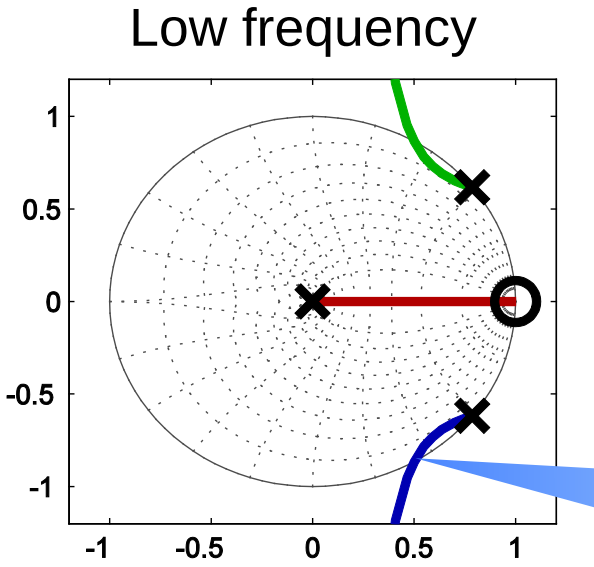
- Design PR controller time constant (T_r)
  - Phase contribution minimal at ω_c

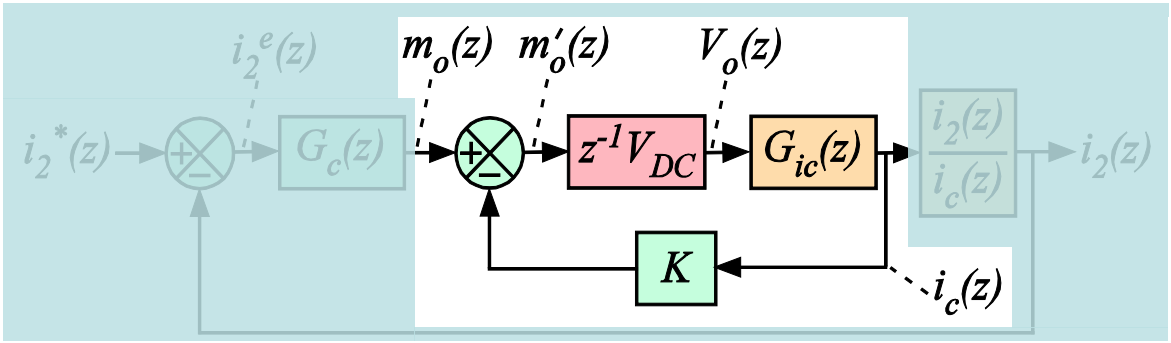
$$T_r \approx 10/\omega_c$$

High Frequency System	
$L_1 = 6mH$	$L_2 = 2mH$
$C_f = 1.5\mu F$	$L_{eq} = 1.5mH$
$2V_{DC} = 650V$	$f_s = 5kHz$
$\omega_{res} = 21.1krads^{-1}$	$T = 0.1msec$
$\omega_{crit} = 10.5krads^{-1}$	$T_d = 0.15msec$
$\varphi_m = 45^o$	$\omega_c = 5.24krads^{-1}$
$K_p = 0.129A^{-1}$	$T_r = 1.9msec$



Inner Loop (i_c) of Active Damping Root Loci

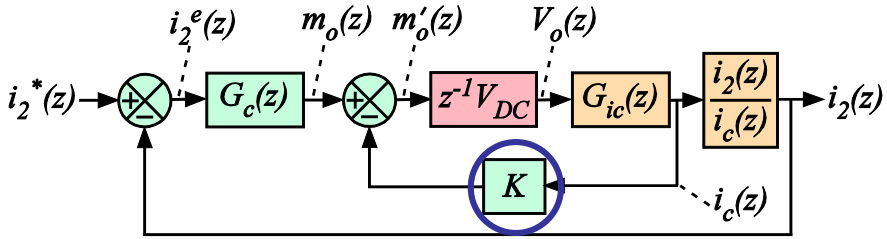




$$z = 0.5 \pm j\sqrt{3}/2$$
Critical frequency

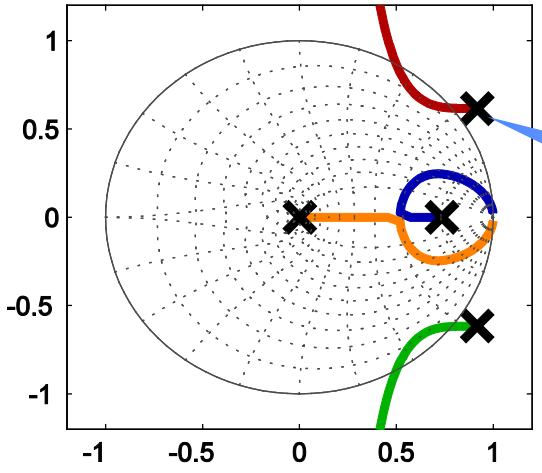
- Only low frequency region examined
  - High frequency → not required
  - Critical frequency → not effective (next slide)
- Provides damping of poles – Limited range of K

Dual Loop Controller Root Loci



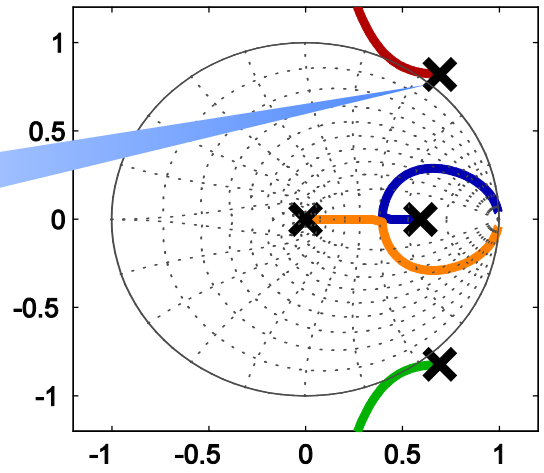
Varying K (active damping)

Low frequency



Poles start unstable

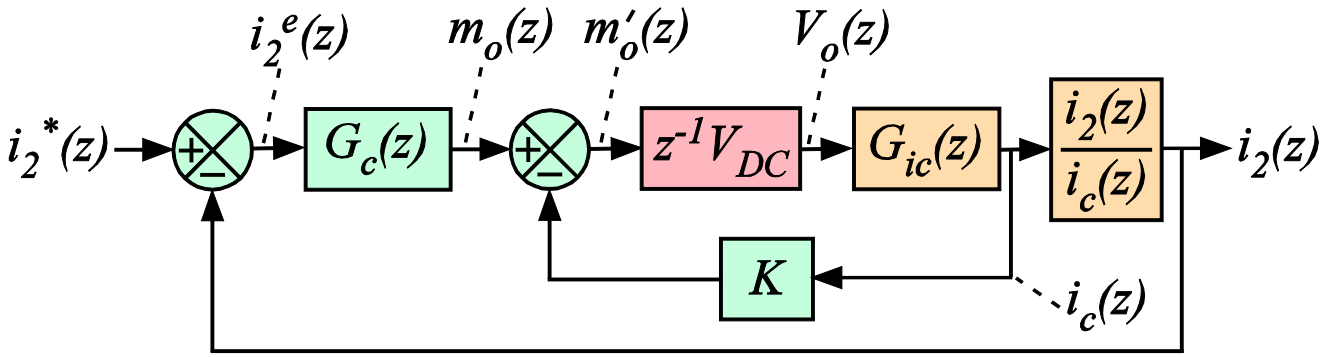
Critical frequency



- At low frequency
  - Poles are moved inside unit circle by active damping
  - Limitation to damping gain K before instability
- At critical frequency – never enter unit circle

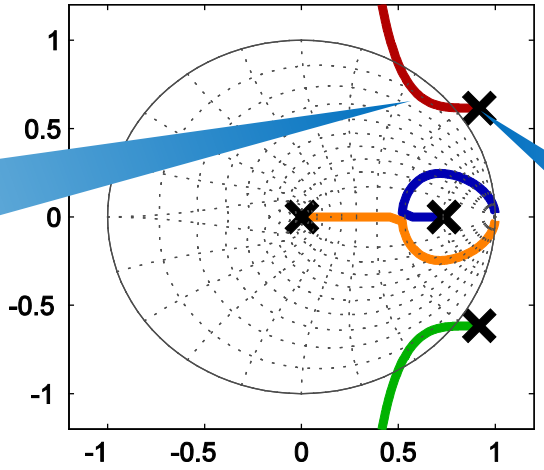
Controller when Resonance Below ω_{crit}

- Dual loop controller (active damping) required
  - Capacitor current feedback selected



- Gain selection will ensure maximally damped poles

Active damping draws poles into stability



Poles originate outside unit circle

Gain Selection when Resonance Below ω_{crit}

- Low frequency still dominated by series filter inductance
- Phase margin must consider phase contribution of resonance
  - Crossover frequency ratio of resonant frequency [Tang et. al. 2012]

$$\omega_c \approx 0.3\omega_{res}$$

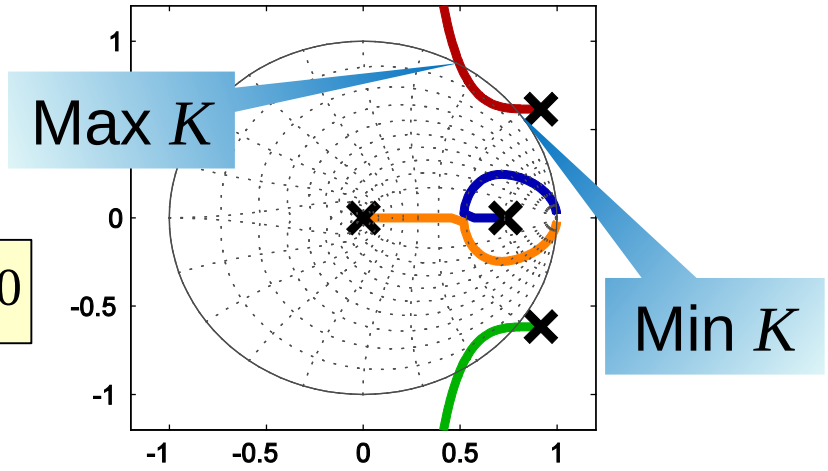
- Proportional gain and PR time constant → Same manner as earlier

$$K_p \approx \frac{\omega_c \left(L_1 + L_2\right)}{V_{DC}} \qquad T_r \approx 10/\omega_c$$

- Best possible damping gain
  - Maximally damped poles
  - Closed loop characteristic equation

$$z + V_{DC} K G_{ic}\left(z\right) + V_{DC} K_P G_{ic}\left(z\right)\left[i_2\left(z\right)/i_c\left(z\right)\right] = 0$$

  - Bounded range for K



Bounded Range for K (Resonance Below ω_{crit})

- **Maximum value (K_{max})**
  - Magnitude of characteristic equation → adapted to equal 1

$$\left| \frac{V_{DC}\sin(\omega_{res}T)}{\omega_{res}L_1} \times \frac{K(z-1)^2 + K_p\gamma_{LC}^2T^2z}{z(z-1)(z^2-2z\cos(\omega_{res}T)+1)} \right| = 1$$

  - Substitute in critical point

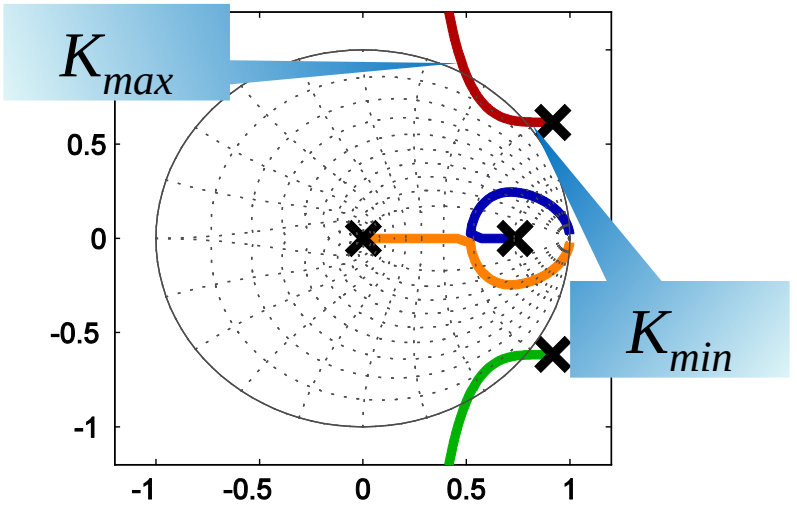
$$z_0 = 0.5 + j\sqrt{3}/2$$

  - Solve for K_{max}

$$K_{\max} = \frac{\omega_{res}L_1}{V_{DC}\sin(\omega_{res}T)}\left|1-2\cos(\omega_{res}T)\right| + K_p\gamma_{LC}^2T^2$$

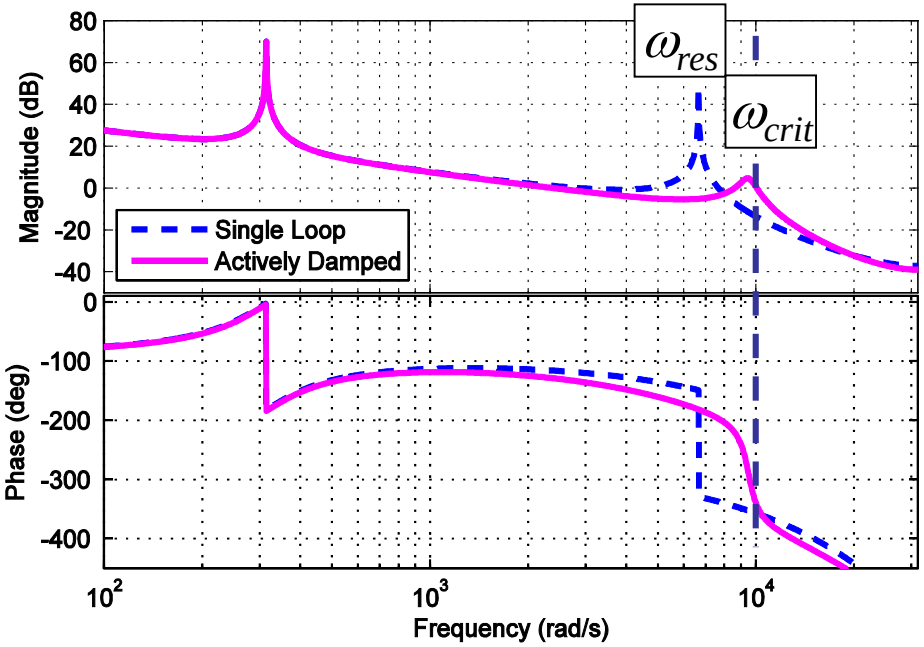
- **Minimum value (K_{min})**
  - Ratio of K to K_p identified using
  - Routh's Stability Criterion [Liu et. al. 2012]
  - Rearranged as limitation

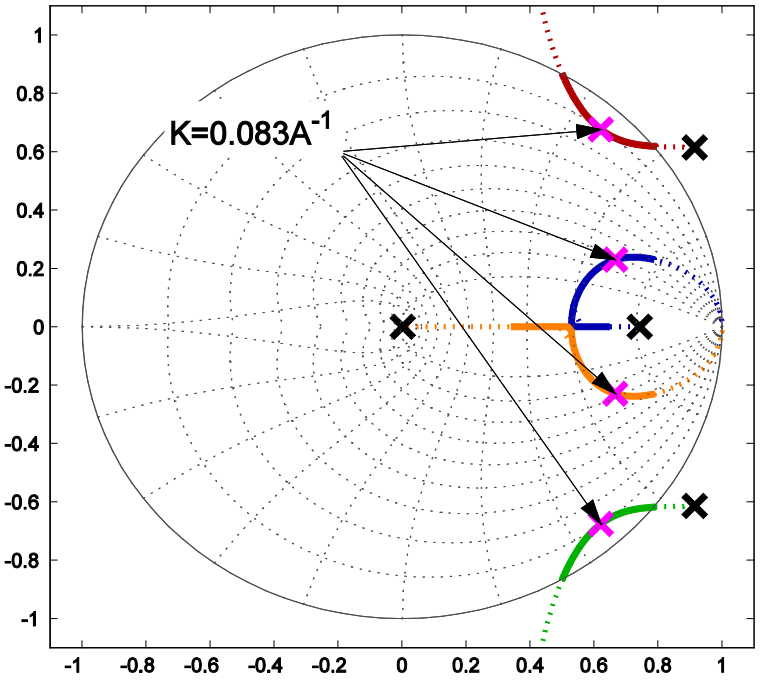
$$\frac{K_p}{K} \le \frac{L_1+L_2}{L_1} \quad \Rightarrow \quad K_{\min} = \frac{K_pL_1}{L_1+L_2}$$



Final Gain Selection (Resonance Below ω_{crit})

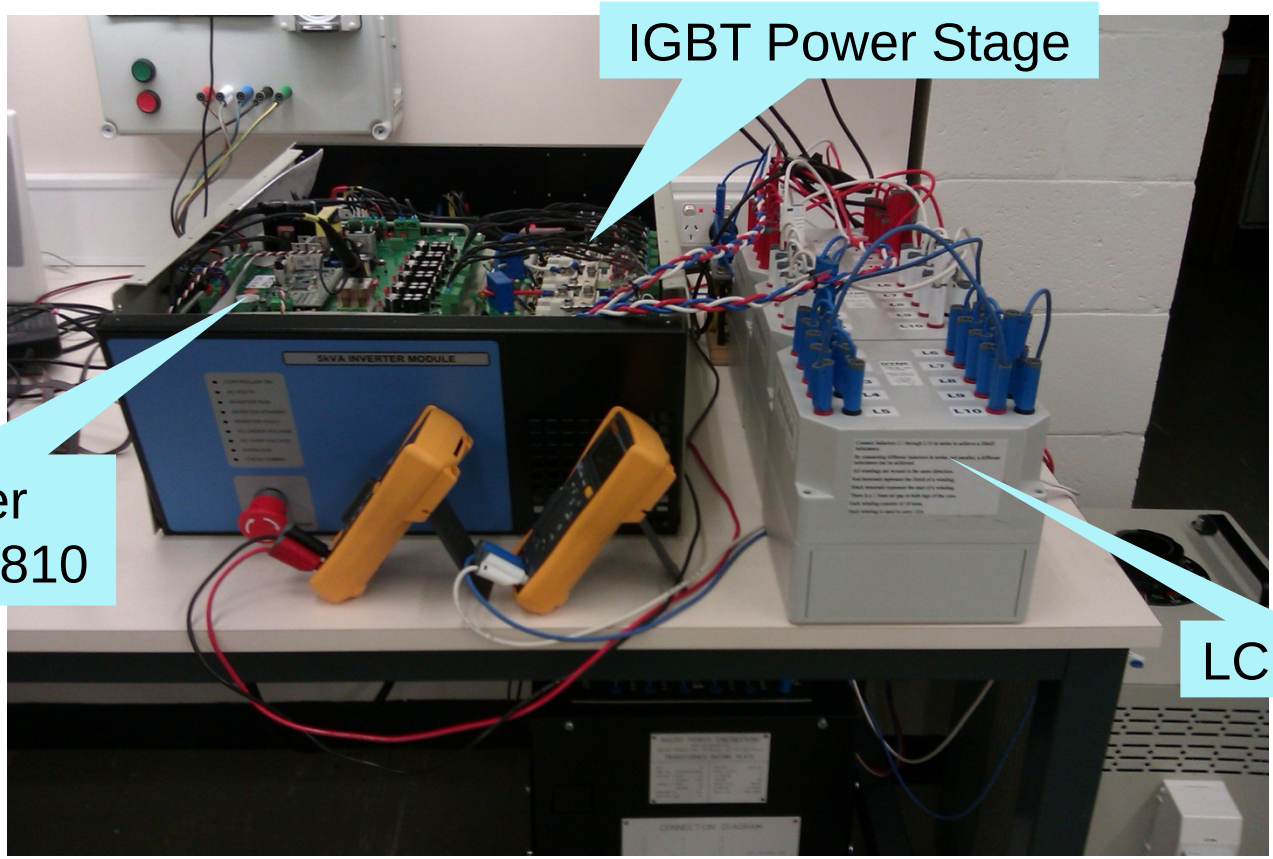
- Bounded range → Solid line
  - $K_{min}=0.044A^{-1}$
  - $K_{max}=0.124A^{-1}$
- Maximum damping $K=0.083A^{-1}$





Low Frequency System		
$L_1 = 6mH$	$L_2 = 2mH$	
$C_f=15\mu F$	$f_s = 5kHz$	
$\omega_{res} = 6.67krads^{-1}$	$\omega_c=0.36\omega_{res}$	
$\omega_{crit}=10.5krads^{-1}$	$\omega_c=2.4krads^{-1}$	
$K_p=0.059A^{-1}$	$T_r=4.2msec$	$K=0.083A^{-1}$

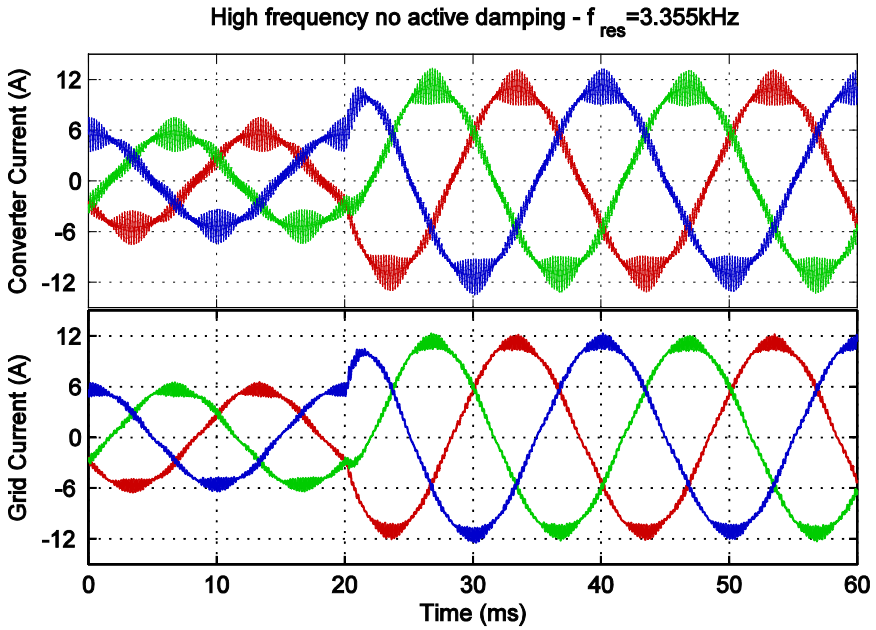
5kVA Experimental Setup



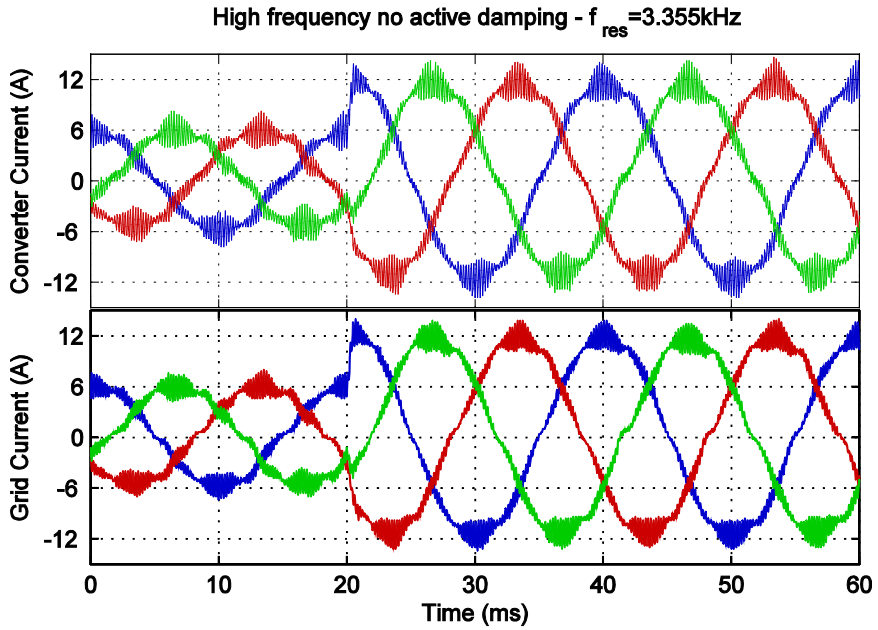
Other System Parameters			
$L_1=6mH$	$L_2=2mH$	$2V_{DC}=650V$	$E=415V_{rms(l-l)}$
$f_{sw}=5kHz$	$f_{samp}=10kHz$	$T=100\mu s$	

High Resonant Frequency System Results

- Simulation (PSIM)



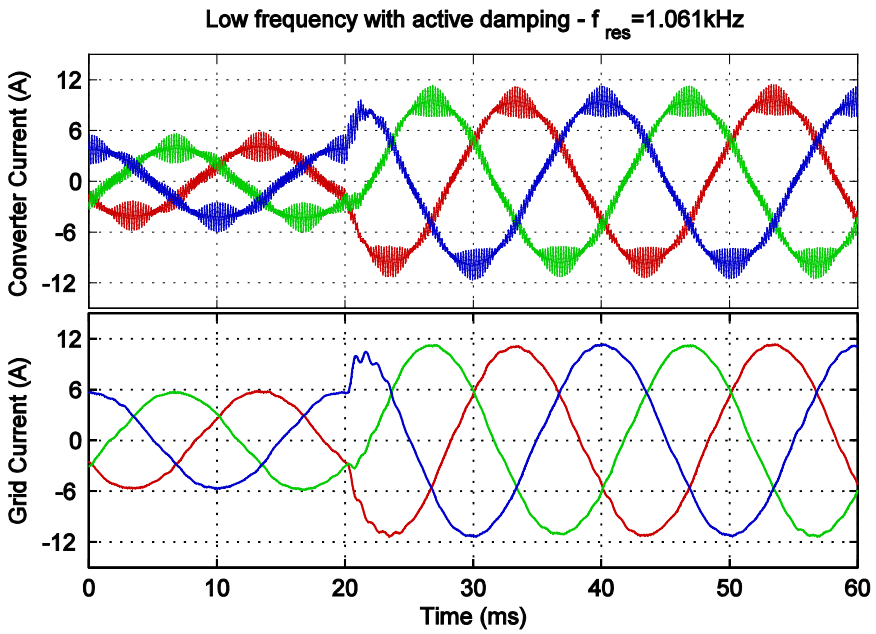
- Experimental



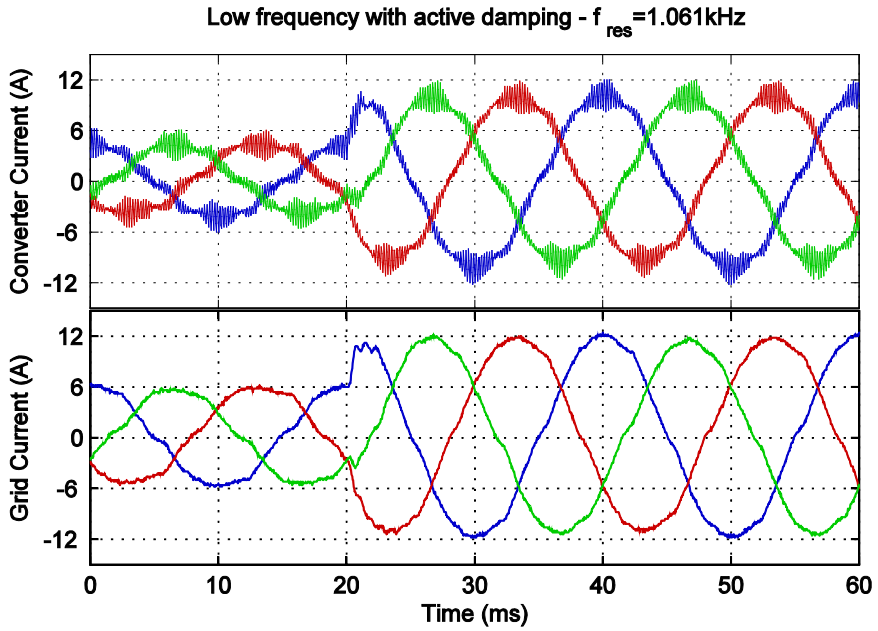
- No visible oscillation
- Rapid transient response → maximised gains
- Similar to single L filter system
- High ripple → poor filtering

Low Resonant Frequency System Results

- Simulation (PSIM)



- Experimental



- Transient damped in 3 to 4 oscillations
  - Agreement with root locus design → underdamped poles
- Minor dynamic performance reduction
  - Limited bandwidth despite maximised gains

Summary of Linear Regulation for LCL Systems

- Discrete time model of actively damped LCL system developed
  - Including sampling and PWM transport delay
- Two distinct regions of operation
  - High resonant frequency
    - Single loop controller
    - Simple gain selection based on L filter
    - Dynamic performance and filtering capacity similar to L filter
  - Low resonant frequency
    - Actively damped controller (dual loop)
    - Root locus gain selection for maximally damped poles
    - Reduced dynamic performance
  - Critical frequency → dual loop active damping ineffective
- Simulation and experimental results validate theoretical models, controller selection and gain calculation